8.231 Physics of Solids I — Fall 2017
Problem Set 1

Posted: Thursday, Sep 7, 2017
Due: **Tuesday, Sep 12, 2017**

Readings (Optional)

- P. W. Anderson. "More is Different". *Science*, **177**, 393 (1972).

Problem 1

AC Conductivity in Drude Theory

Consider an alternating electric field of frequency ω is applied to the metal, where $\mathbf{E} \sim e^{-i\omega t}$. It will induce current $\mathbf{J} \sim e^{-i\omega t}$. Modify the calculation for DC conductivity of Drude model to obtain an expression for the complex AC conductivity $\sigma(\omega)$. For small ω, your results should reproduce the low energy trend in Figure 1.

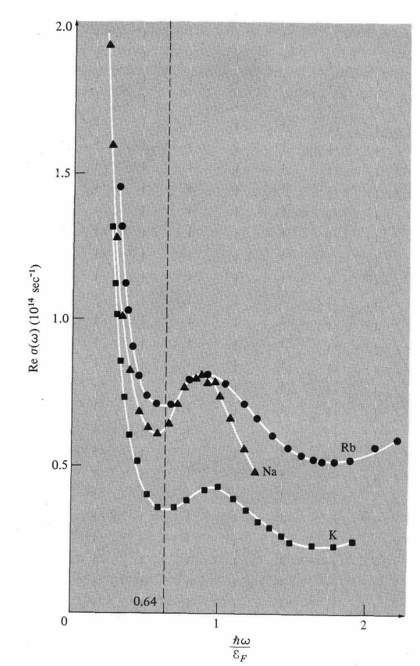

Figure 1: Real part of the optical conductivity in three alkali metals, deducing from reflectivity measurements. E_F is the free electron Fermi energy. The higher energy features above $0.64E_F$ are not relevant for this problem, and will be discussed later in this course.

Problem 2

Electromagnetic Waves in Metals

(a) Neglecting bounded charges and currents, the Maxwell's equations in metals are given by

$$\nabla \cdot \mathbf{E} = 0, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right), \tag{4}$$

Assume Ohm's law $\mathbf{J} = \sigma \mathbf{E}$. Derive the wave equation for the electric field $\mathbf{E}$. Solve the complex wave vector $\tilde{k}$ for the given frequency ω for a plane wave in the infinity space.

(Hint: $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$. Solve the wave equation with ansatz $\mathbf{E} = \mathbf{E}_0 e^{i(\tilde{k}z - \omega t)}$.)

(b) Using $\sigma(\omega)$ from Problem 1, compute $\tilde{k}$ as a function of ω. Simplify your result in the high-frequency limit of $\omega\tau \gg 1$. You should find there exists a critical frequency ω_p so that $\tilde{k}$ is real when $\omega > \omega_p$ and is imaginary when $\omega < \omega_p$. ω_p is called the plasma frequency. Identify ω_p. What are the physical consequences when ω is above or below ω_p?