8.231 Physics of Solids I — Fall 2017
Problem Set 7

Posted: Tuesday, Oct 24, 2017
***Due:* Tuesday, Oct 31, 2017**

Note: Bonus problems are more challenging. It is fine you do not solve them. By solving them you can earn extra credits.

Problem 1

de Haas-van Alphen Effect

In Problem 1 of Problem Set 3, we computed the magnetization M of free electrons as a function of magnetic field B by considering only the spin effect. In this problem, we will consider the orbital effect.

Under strong magnetic field, the magnetization in metals oscillates periodically with $1/B$. Historically, this phenomenon was discovered in 1930 and is known as de Haas-van Alphen effect.

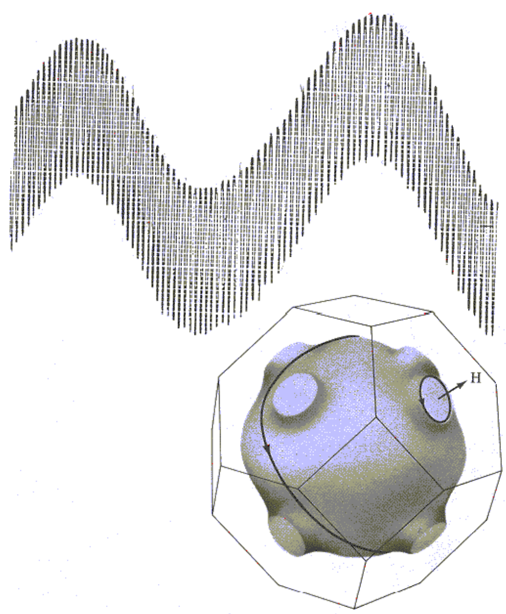

Figure 1: de Hass-van Alphen effect of silver.

In fact, not only the magnetization oscillates with magnetic field, but other quantities like resistivity and specific heat oscillate as well. All these phenomena are referred as "quantum oscillation". As we will see in this problem, these quantum oscillations can be used to infer the shape of the Fermi surface. To understand the essential physics of quantum oscillation, we consider free electrons of mass m in two dimensions without spin degeneracy.

(a) Suppose a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ is perpendicular to the two-dimensional electron plane. The electrons form Landau levels. Write down directly the energy eigenvalue and the degeneracy for each Landau level.

(Hint: You may want to recall Recitation Note 5.)

(b) At zero temperature $T = 0$, fix the chemical potential and compute M as a function B. Sketch M as a function of $1/B$. You should find that M oscillates periodically with $1/B$. Identify the period of the oscillation, and express it with the Fermi surface area.

(Hint: Use grand potential $\Phi = E - TS - \mu N$ and $M = -(\partial\Phi/\partial B)_\mu/V$. Landau levels are always fully filled.)

(c) At zero temperature $T = 0$, fix the total electron number and compute M as a function B. Sketch M as a function of $1/B$. You should also find that M oscillates periodically with $1/B$. What is the difference between this result and the result in (b).

(Hint: Use Helmholtz potential $A = E - TS$ and $M = -(\partial A/\partial B)_N/V$. Landau levels are not always fully filled.)

(d) Explain the physical mechanism behind the oscillation in (b) and (c) separately. If the electron dispersion is distorted from quadratic, in which scenario the oscillation period in $1/B$ is still strict?

(e) (Bonus) At finite temperature $T \ll T_F$, fix the chemical potential and compute M as a function B. How to interpret your result?

You will find this Fourier transform useful:

$$\int_{-\infty}^{\infty} dz \frac{e^{i\lambda z}}{1+\cosh(bz)} = \frac{2\pi\lambda}{b^2}\frac{1}{\sinh(\pi\lambda/b)}. \tag{1}$$