8.231 Physics of Solids I

Course Summary

Massachusetts Institute of Technology

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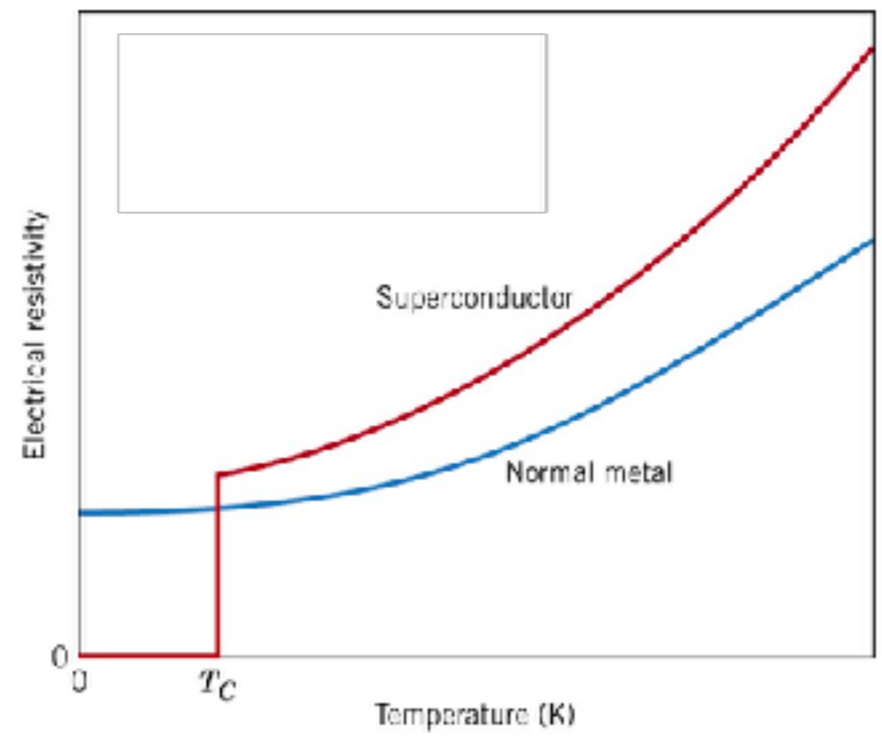
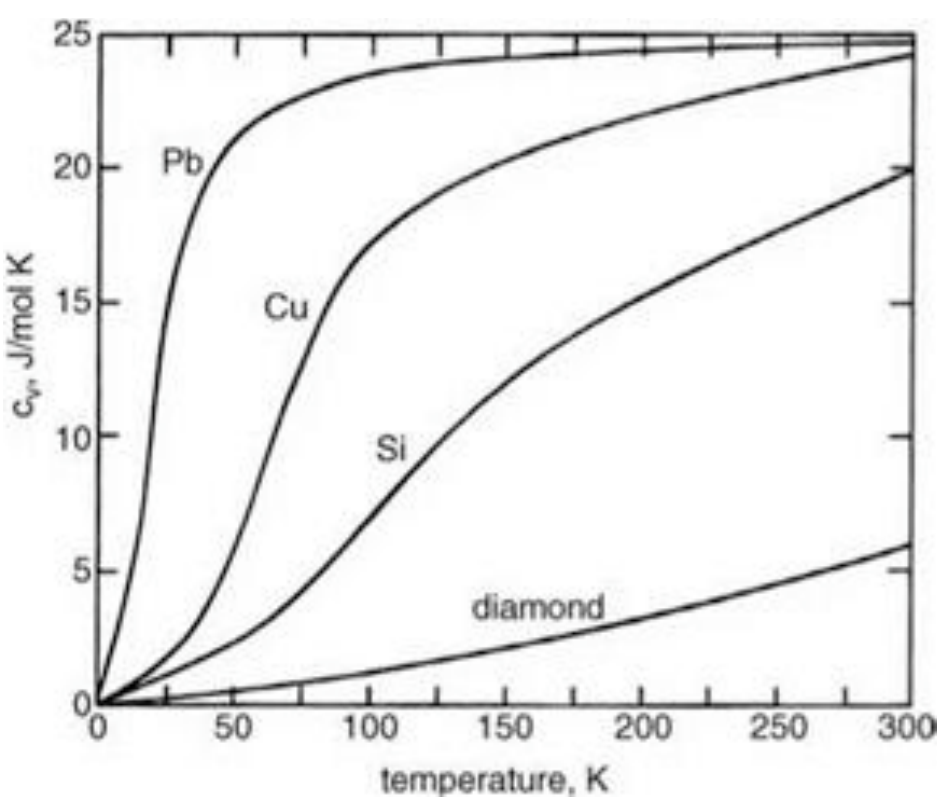
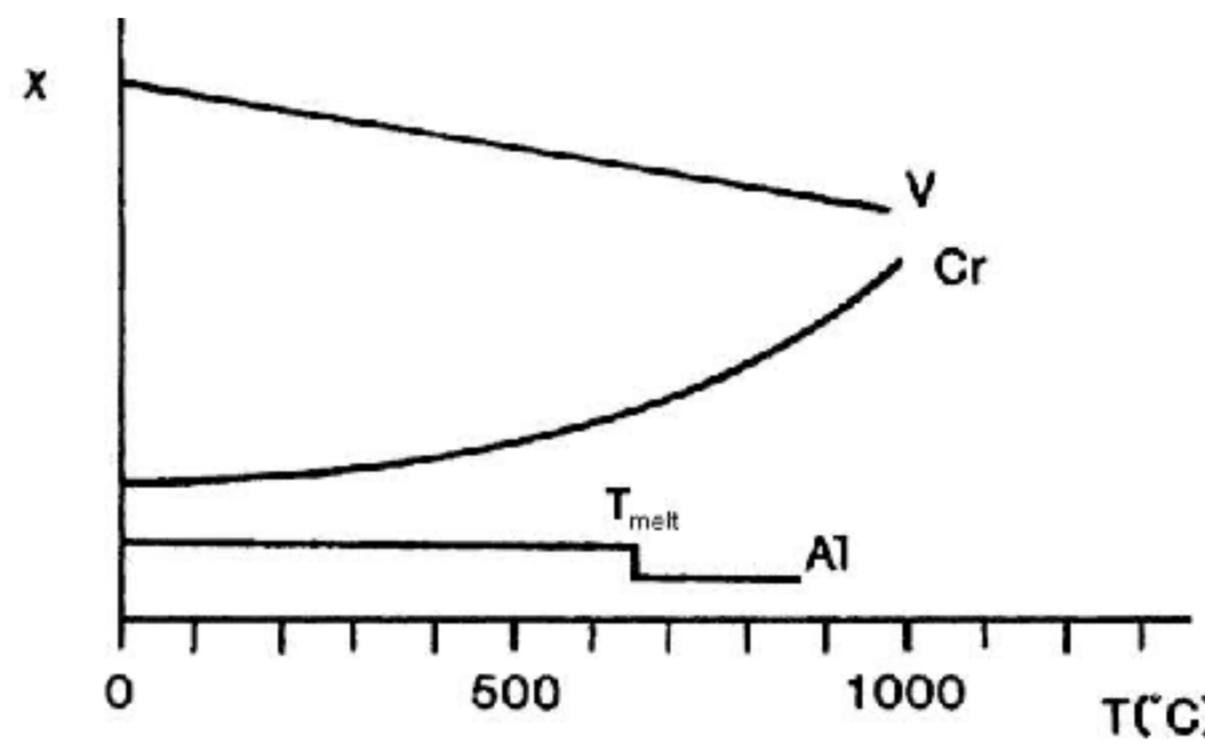
12/12/17

Normal Metals at Low T

- Electrical resistivity
- Heat capacity
- Magnetic susceptibility







Drude Model (1900)

Assumptions:

- No el-ph interaction
- No el-el Coulomb interaction, only collisions characterized by τ
- Electrons are classical, obey Maxwell-Boltzmann dist.

$$\frac{d\mathbf{p}}{dt} = -e\left(\mathbf{E} + \frac{\mathbf{p}}{m} \times \mathbf{B}\right) - \frac{\mathbf{p}}{\tau}$$

Drude Model (1900)

Successes:

- DC Electrical Conductivity $\sigma_0 = \frac{ne^2\tau}{m}$
- AC Electrical Conductivity (Pset 1) $\sigma = \frac{\sigma_0}{1 - i\omega\tau}$
- Hall Conductivity $R_H \equiv \frac{E_y}{BJ_x} = -\frac{1}{ne}$
- Thermal Conductivity (Rec 1) $\kappa = \frac{1}{3}cmv^2$
- Widemann-Franz Law $\frac{\kappa}{\sigma T} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2$ (by chance!)

Sommerfeld Model (1928)

Maxwell-Boltzmann dist. ⟶ Fermi-Dirac dist.

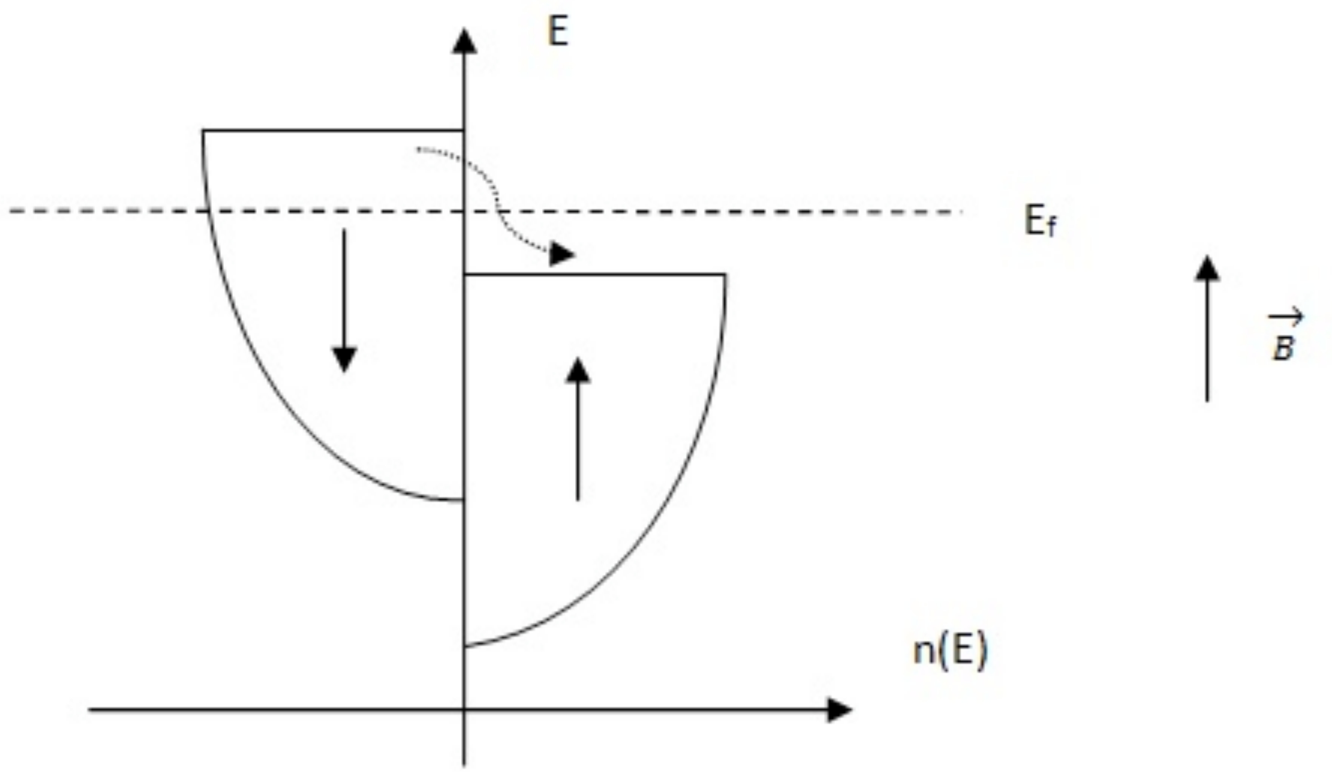
$$\kappa = \frac{1}{3} c m v^2$$

	c	v
M-B	$\frac{3}{2} n k_B$	$\frac{mv^2}{2} = \frac{3k_B T}{2}$
F-D	$\sim D(E_F) k_B T$	$\frac{mv_F^2}{2} = E_F$

Sommerfeld Model (1928)

Successes:

- Linear Heat Capacity $c \sim k_B T D(E_F)$
- Pauli Paramagnetism (Pset 3) $\chi \equiv \frac{\partial M}{\partial H} = \mu_B^2 D(E_F)$



Density of States

Density of states at the Fermi energy is the "DNA" of metals!

- Heat Capacity $c \sim k_B T D(E_F)$
- Pauli Paramagnetism (Pset 3) $\chi \equiv \frac{\partial M}{\partial H} = \mu_B^2 D(E_F)$
- Binding Energy of Cooper Pair (Pset 8)

$$E_B \sim \hbar\omega_D \exp\left(\frac{2}{gD(E_F)}\right)$$

Normal Metals at Low T

- Electrical resistivity

impurity el-el (Fermi Liquid)

$$\rho(T) = \begin{cases} \rho_0 + \gamma T^2 + \beta T^5, & T < T_D, \\ \alpha T, & T > T_D. \end{cases}$$

el-ph

- Heat capacity

$$c(T) = \begin{cases} \gamma T + \beta T^3, & T < T_D, \\ \frac{3}{2} k_B, & T > T_D. \end{cases}$$

- Magnetic susceptibility

$$\chi(T) \sim \text{const.}$$

Periodic Potential

How to deal with the periodic structure?

Fourier transform!

Reciprocal Lattice

- (Mathematically) Constitute all possible Fourier components;

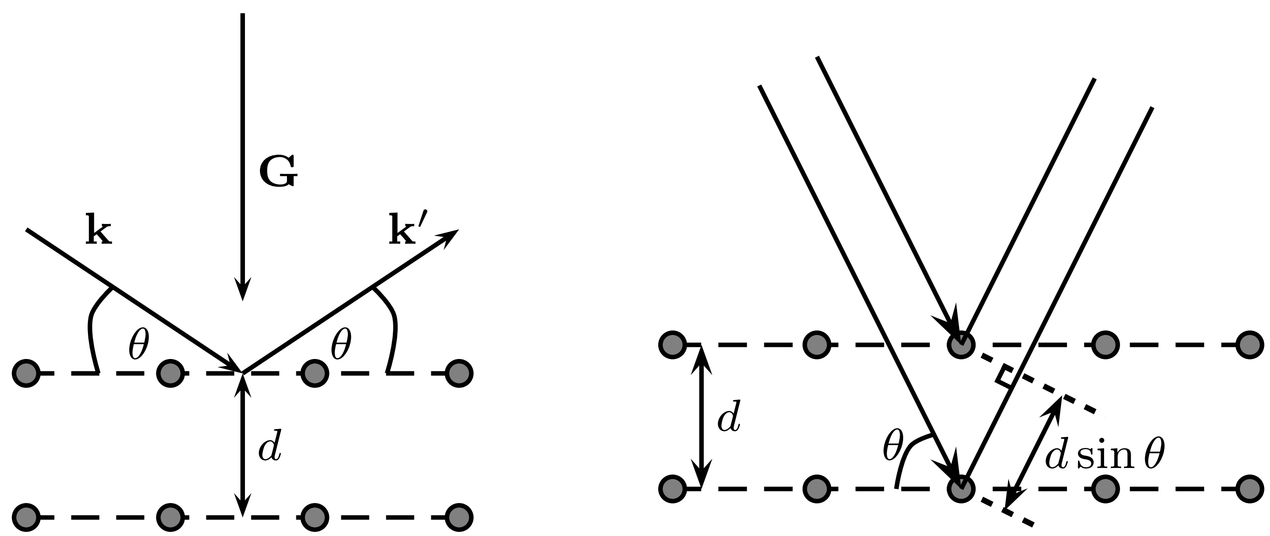
$$V(\mathbf{r}) = \sum_{\mathbf{G}} V(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}, \;\; \mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3$$

- (Geometrically) Specify all lattice planes;

- (Physically) Specify all possible X-ray diffractions through conservation of crystal momentum.

(Rec 4)

X-ray Diffraction (XRD)



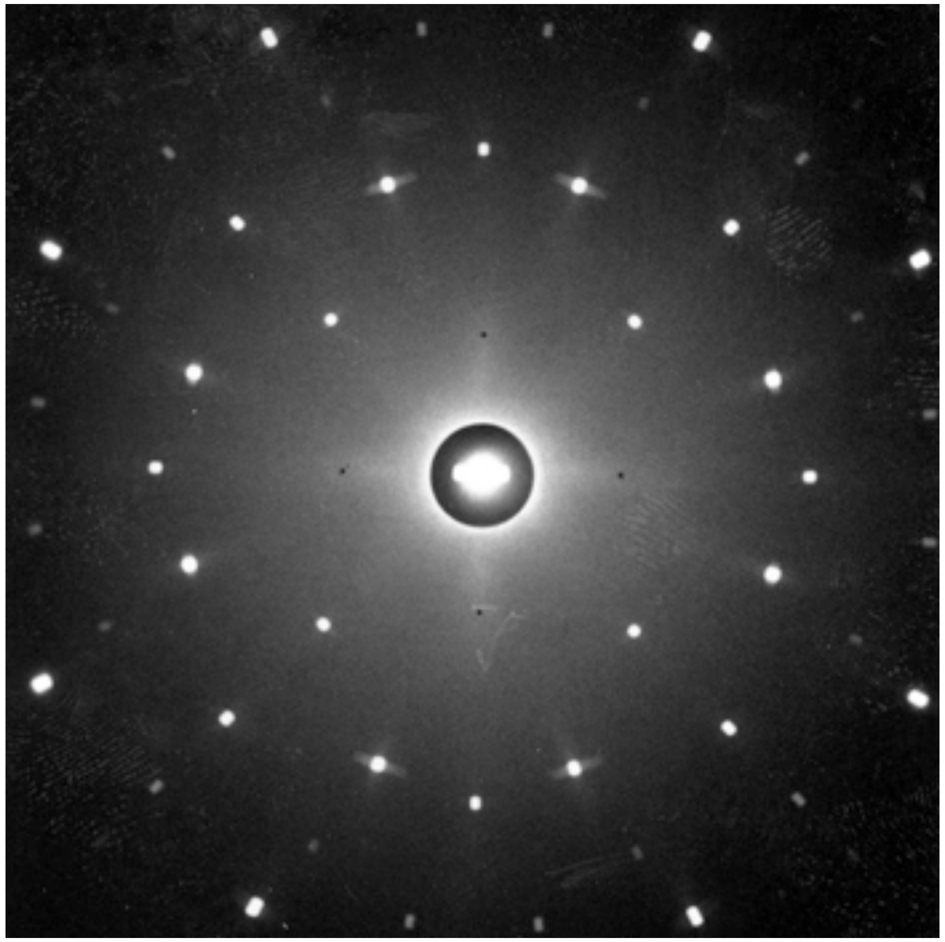
- Laue condition (conservation of crystal momentum)

$$\Delta \mathbf{k} = \mathbf{G}$$

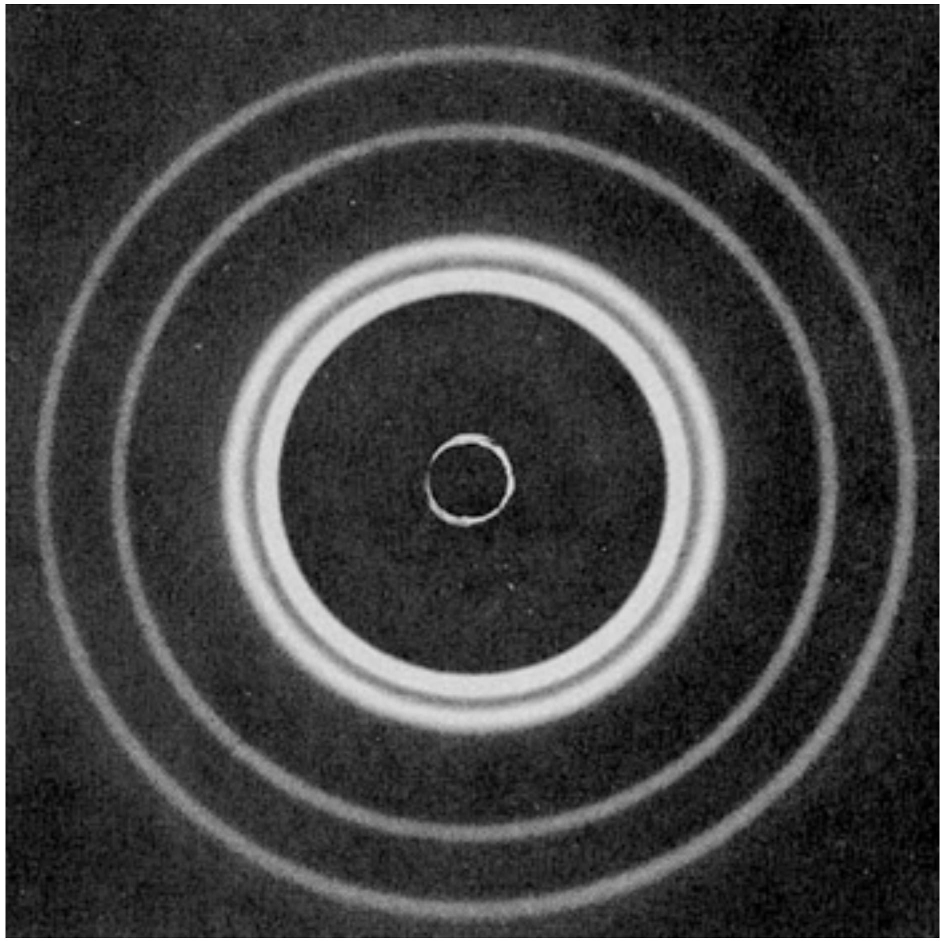
- Bragg condition (elastic scattering) $2d \sin\theta = n\lambda$

(Rec 4)

X-ray Diffraction (XRD)



single crystal



powder

Bloch Theorem

$$\left(-\frac{\hbar^2\nabla^2}{2m}+V(\mathbf{r})\right)\Psi = E\Psi \qquad V(\mathbf{r}+\mathbf{R}) = V(\mathbf{r})$$

“When I started to think about it, I felt that the main problem was to explain how the electrons could sneak by all the ions in a metal.... By straight Fourier analysis I found to my delight that the wave differed from the plane wave of free electrons only by a periodic modulation. ”

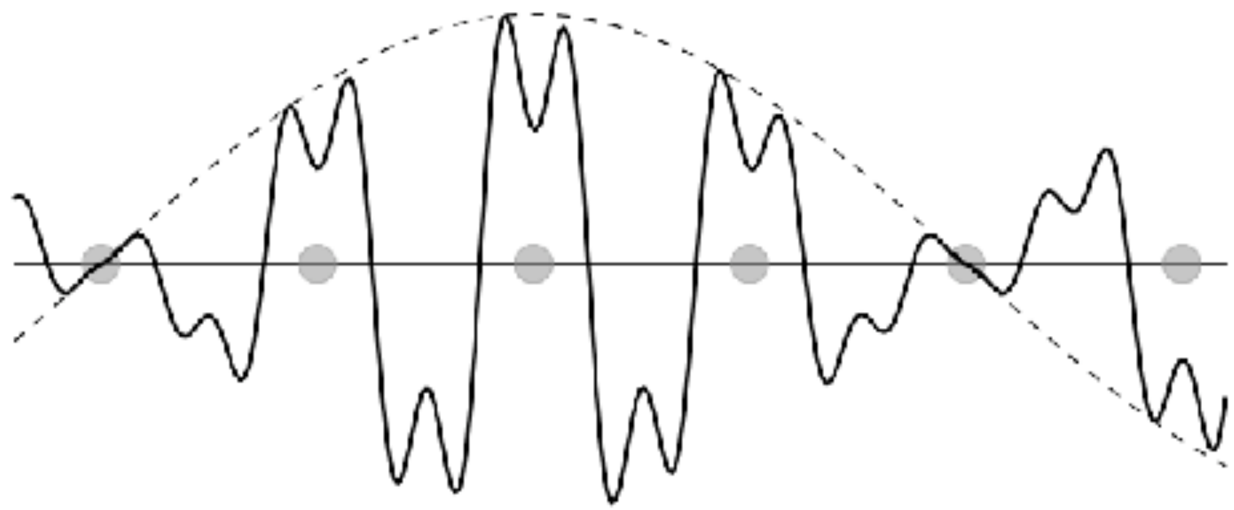
$$\Psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n\mathbf{k}}(\mathbf{r}), \quad u_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r}+\mathbf{R})$$

$$H\Psi_{n\mathbf{k}} = E_n(\mathbf{k})\Psi_{n\mathbf{k}}, \quad E_n(\mathbf{k}) = E_n(\mathbf{k}+\mathbf{G})$$

Band Structure

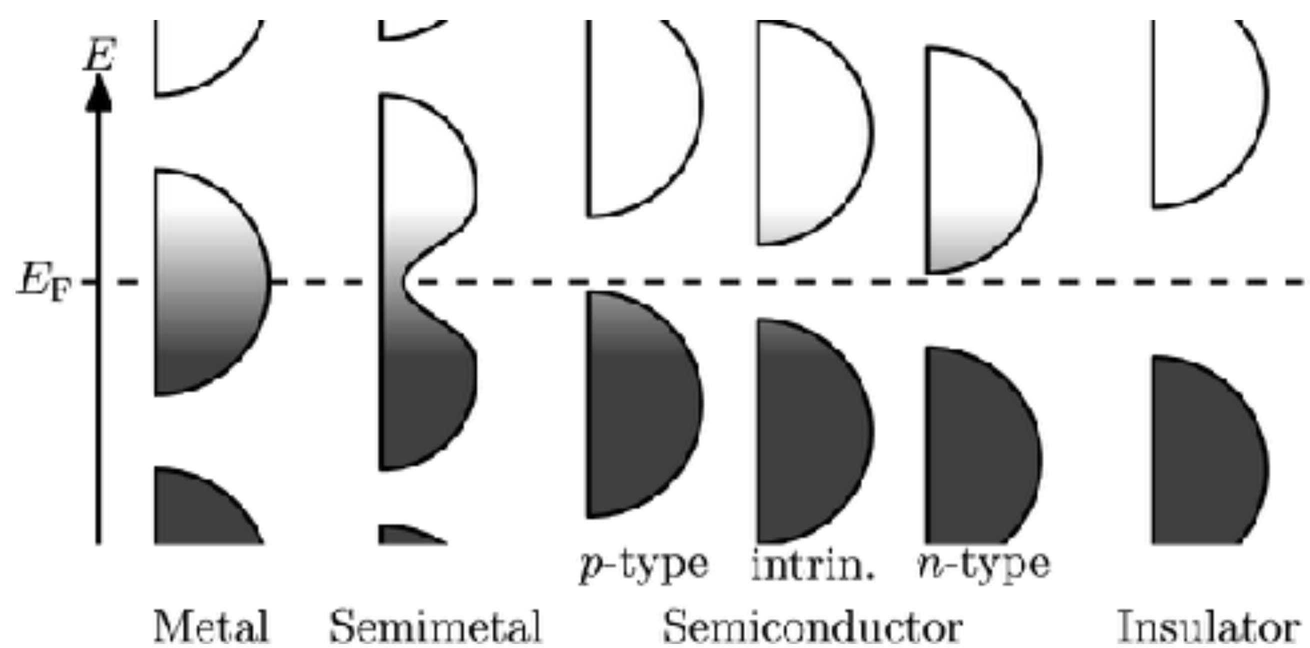
$$\Psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n\mathbf{k}}(\mathbf{r})$$

$$u_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r}+\mathbf{R})$$



$$H\Psi_{n\mathbf{k}} = E_n(\mathbf{k})\Psi_{n\mathbf{k}}$$

$$E_n(\mathbf{k}) = E_n(\mathbf{k}+\mathbf{G})$$



Band Structure

- Exact Solvable Models
  Kronig–Penney model (1D periodic square potential)

- Perturbative Models

$$\left(-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r})\right)\Psi = E\Psi$$

  - Tight-binding model (zero kinetic energy limit)

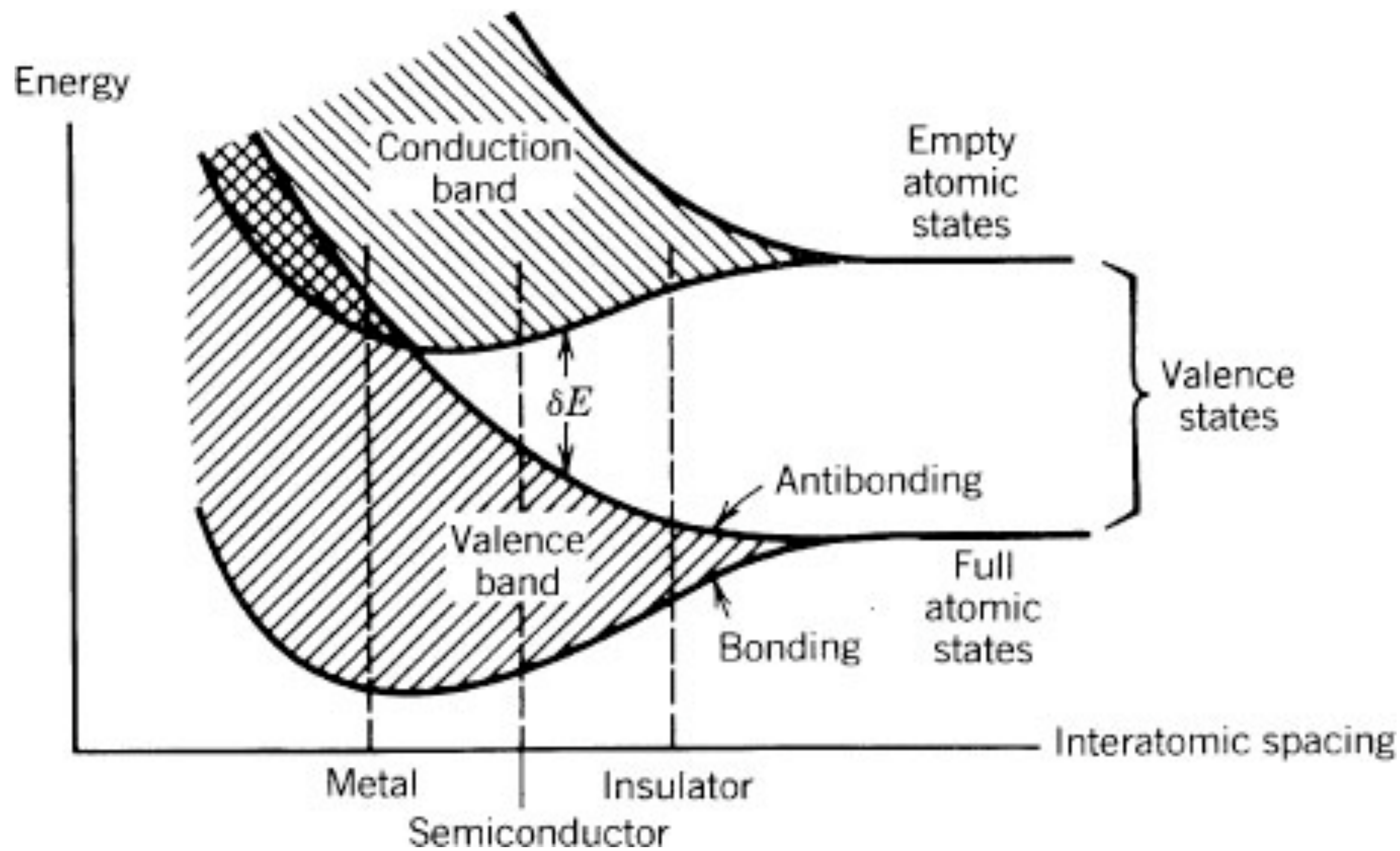
  - Nearly-free electron model (zero potential energy limit)

Tight-binding Model

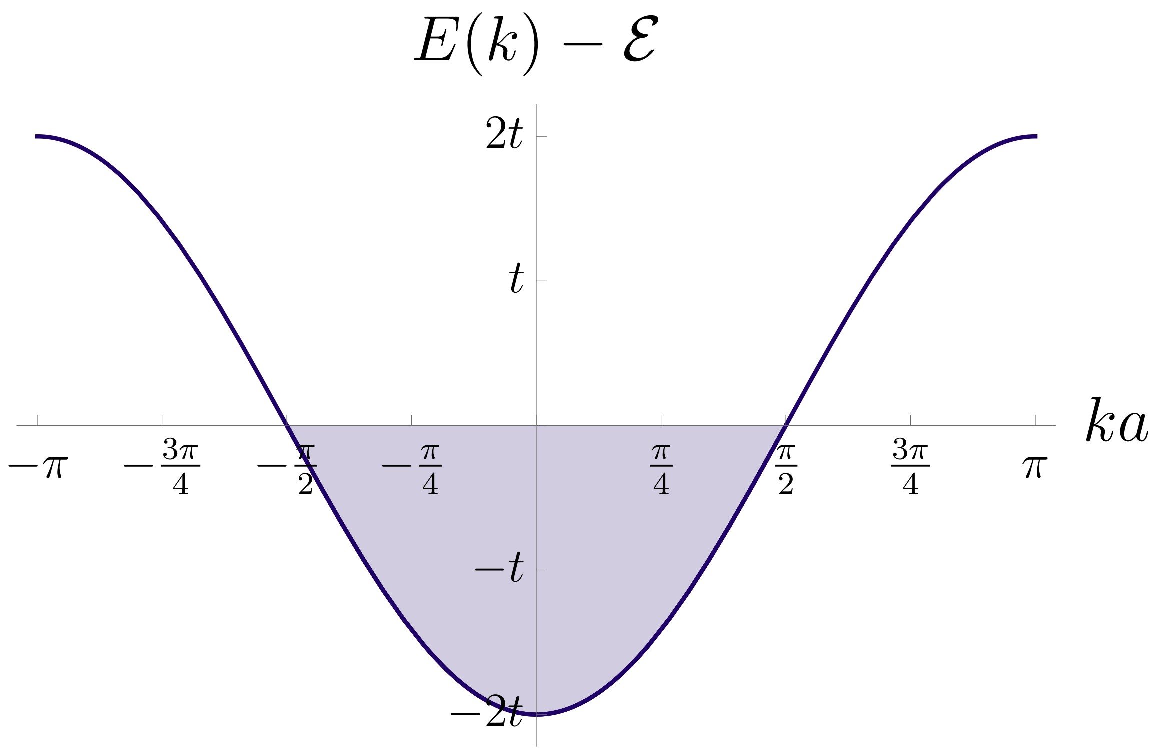
- Assume weak kinetic energy, treated perturbatively

- Gradually decrease interatomic spacing

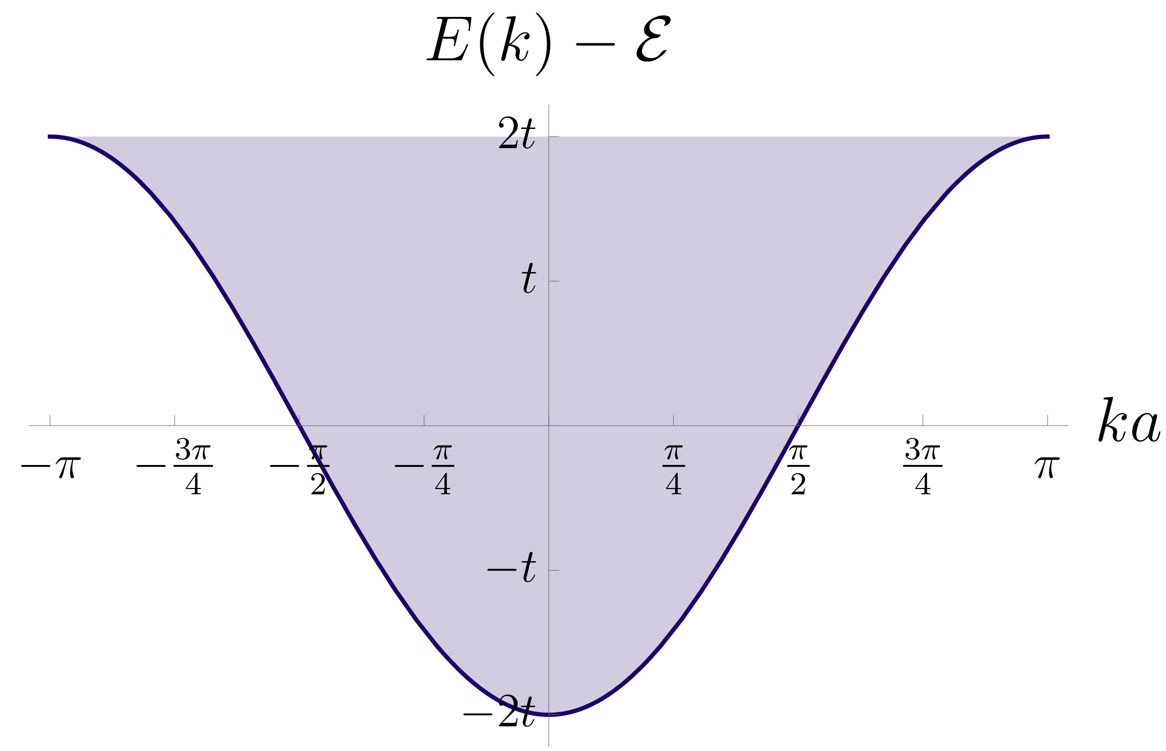
$$E(\mathbf{k}) = \varepsilon_0 - t \sum_{\mathrm{NN}} e^{i\mathbf{k}\cdot\mathbf{R}}$$



Tight-binding Model

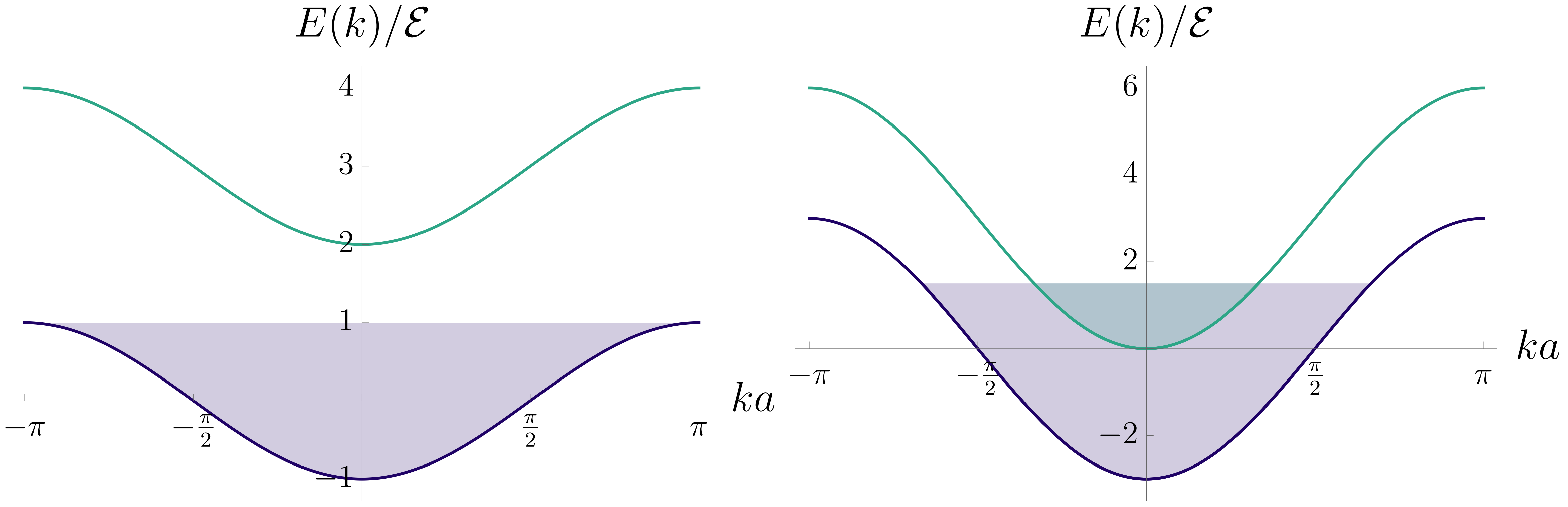


monovalent
metal



divalent
insulator

Tight-binding Model



Nearly-free Electron Model

- Assume weak periodic potential, treated perturbatively
- Works well for good metals (IA, IIA)

Non-degenerate perturbation theory:

$$E(k) = E^0(k) + \langle k|V(x)|k\rangle + \sum_{k'} \frac{|\langle k|V(x)|k'\rangle|^2}{E^0(k) - E^0(k')}$$

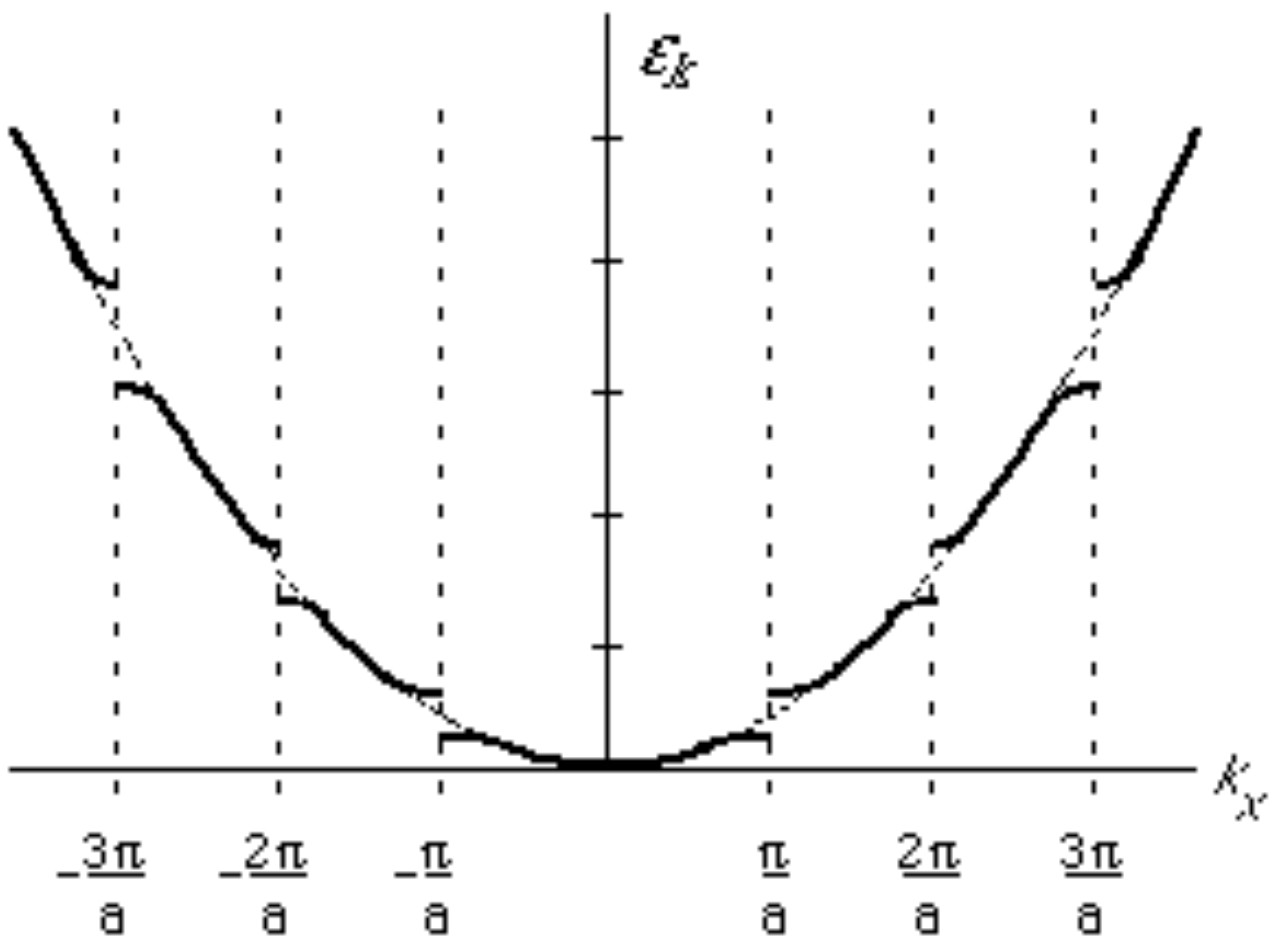
Near the boundary, needs degenerate perturbation theory:

depends on
$\Delta k \equiv 2\pi/L$

$$E(k_0) = E^0(k_0) \pm |V(G)|$$

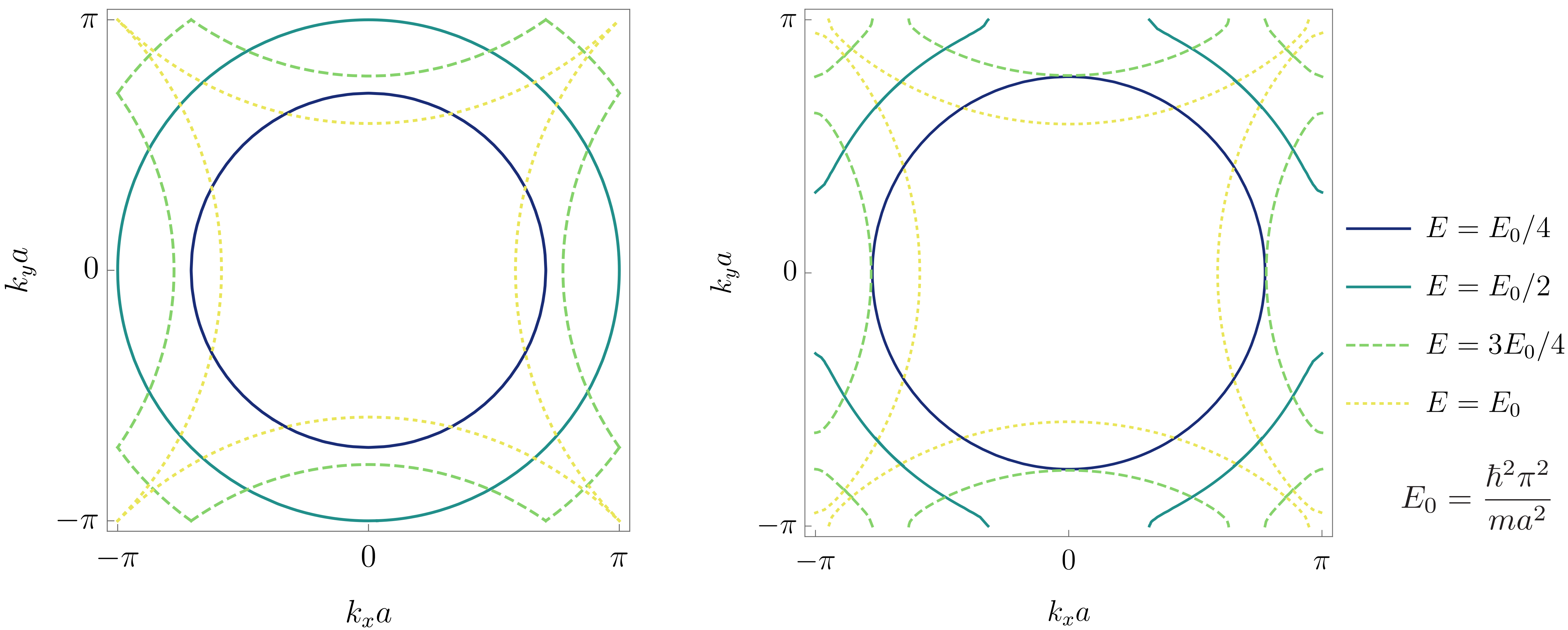
Nearly-free Electron Model

$$E(k_0) = E^0(k_0) \pm |V(G)|$$



Nearly-free Electron Model

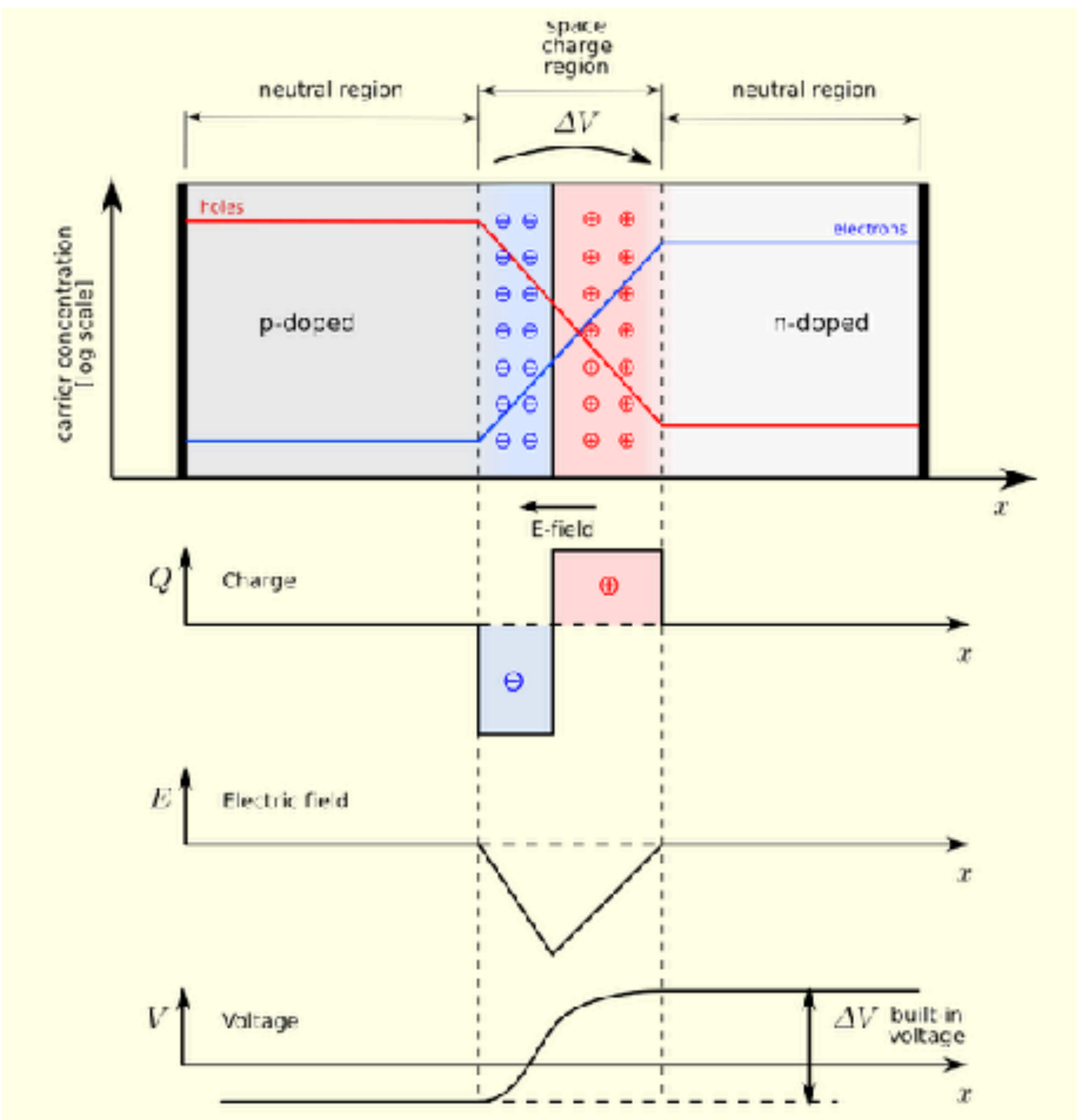
$$V(\mathbf{r}) = -V_0 \left[\cos\left(\frac{2\pi x}{a}\right) + \cos\left(\frac{2\pi y}{a}\right)\right]$$

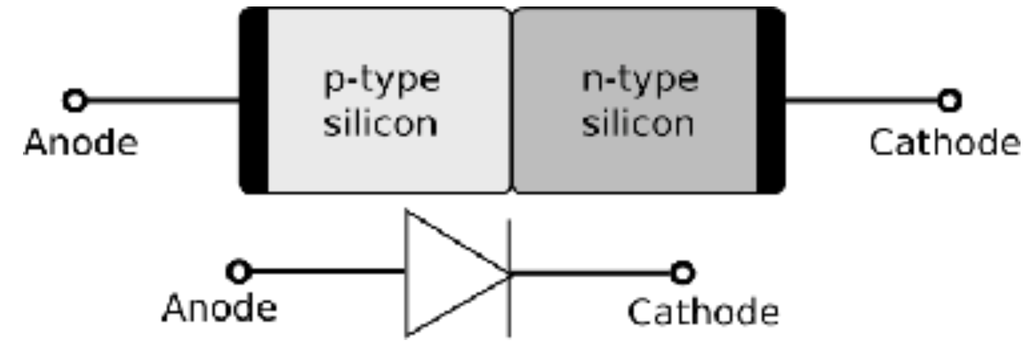


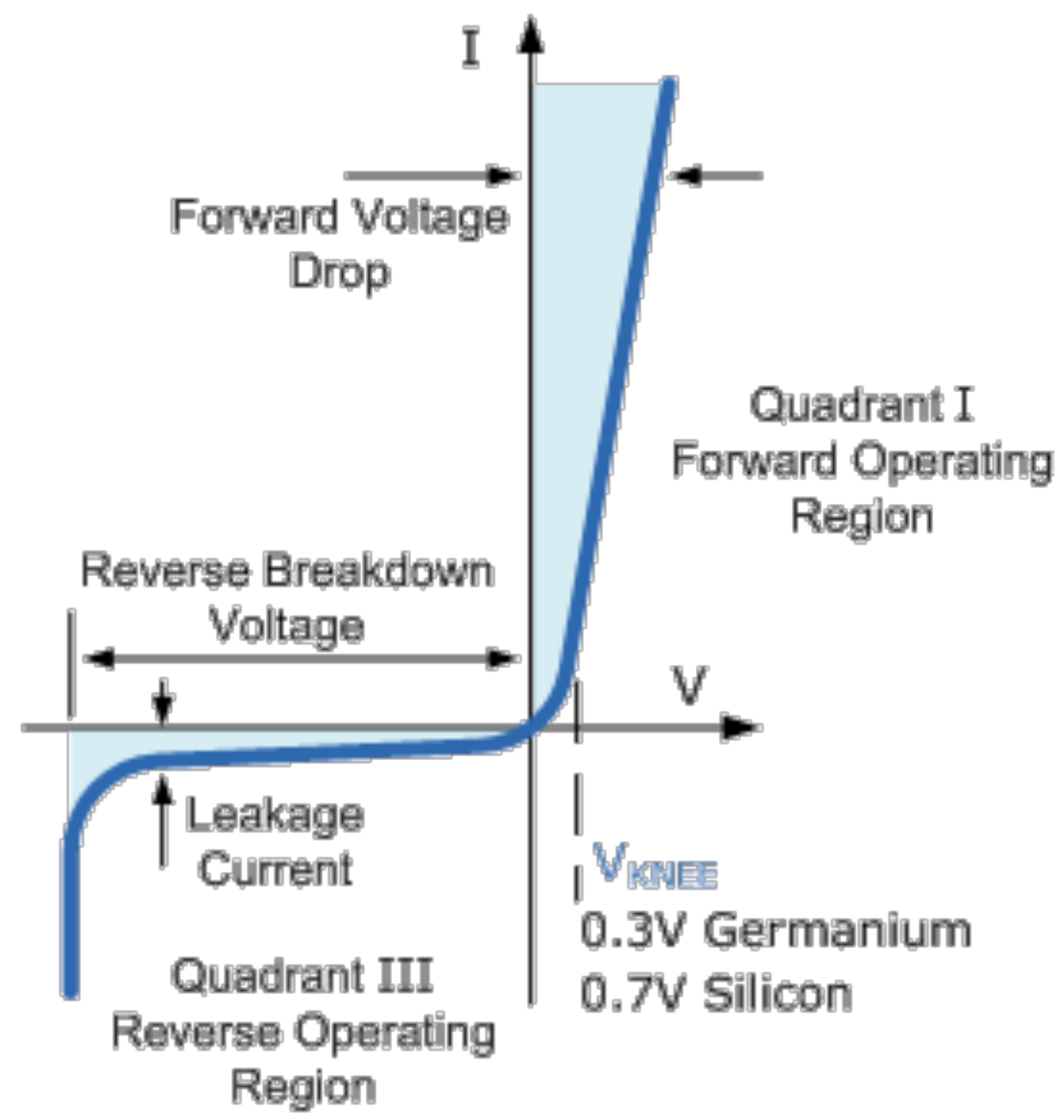
$V_0 = 0$ small V_0

Applications of Band Theory

- p-n junction: diode

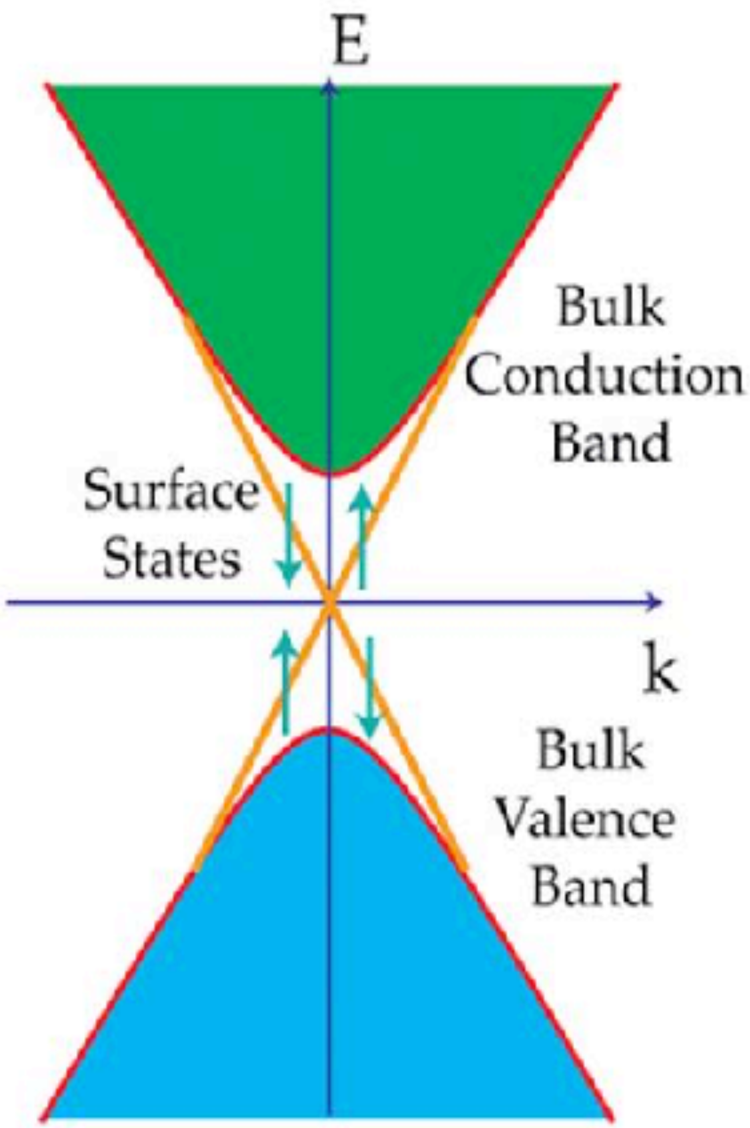


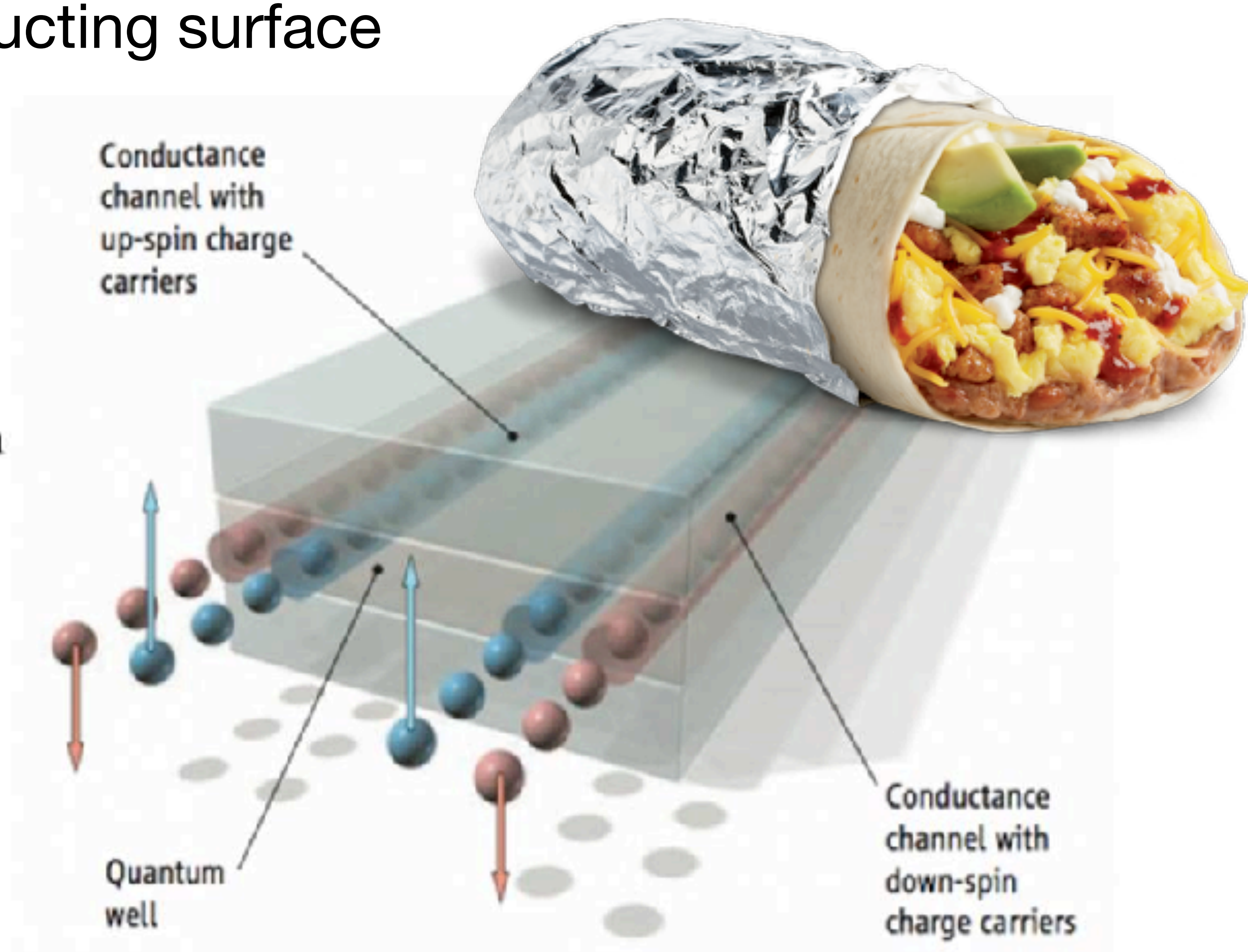




Topological Insulators

Insulating bulk, Conducting surface





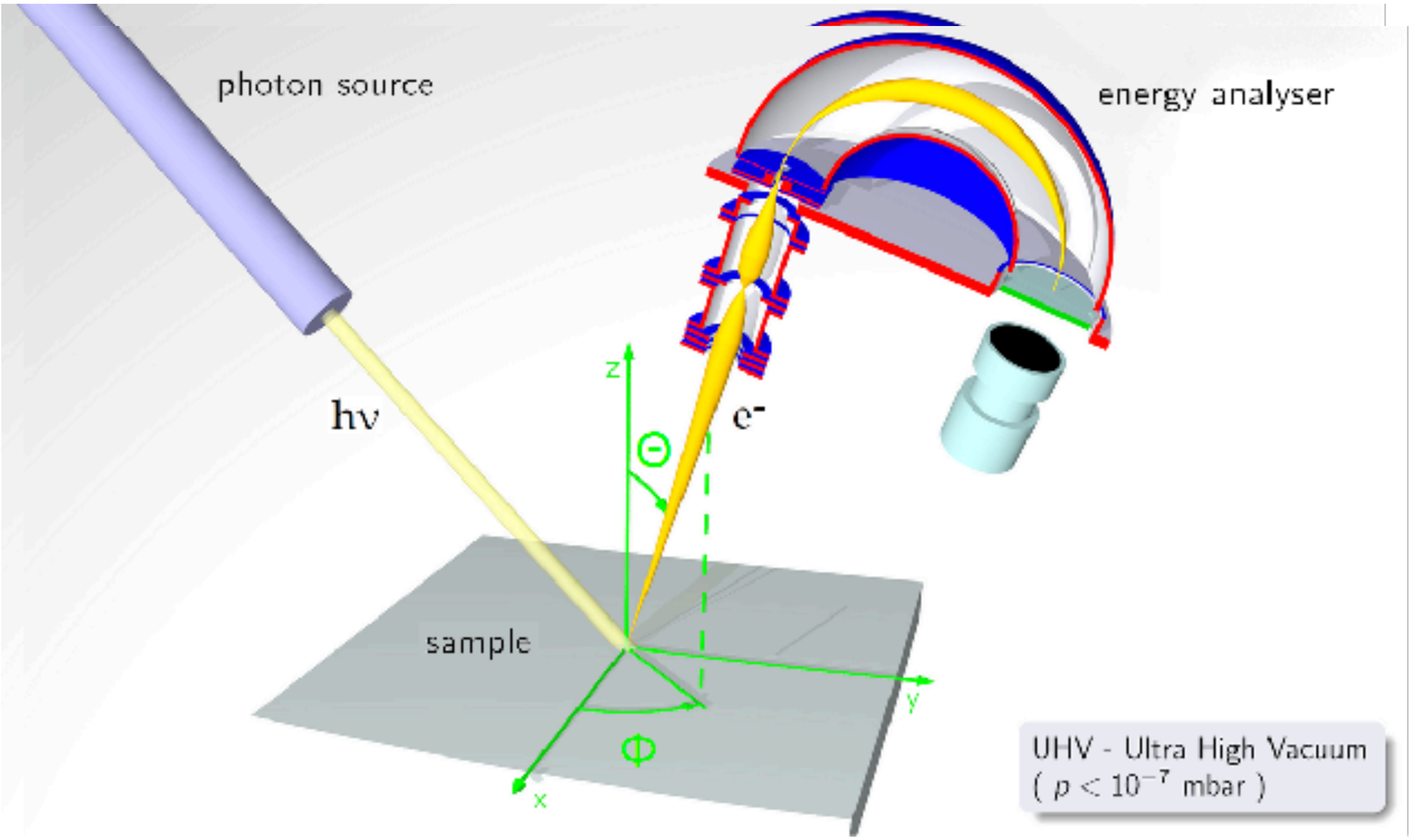
Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

(Rec 6, 9)

Band Structure Measurement

- Angle-resolved photoemission spectroscopy (ARPES)
- Quantum oscillations

ARPES



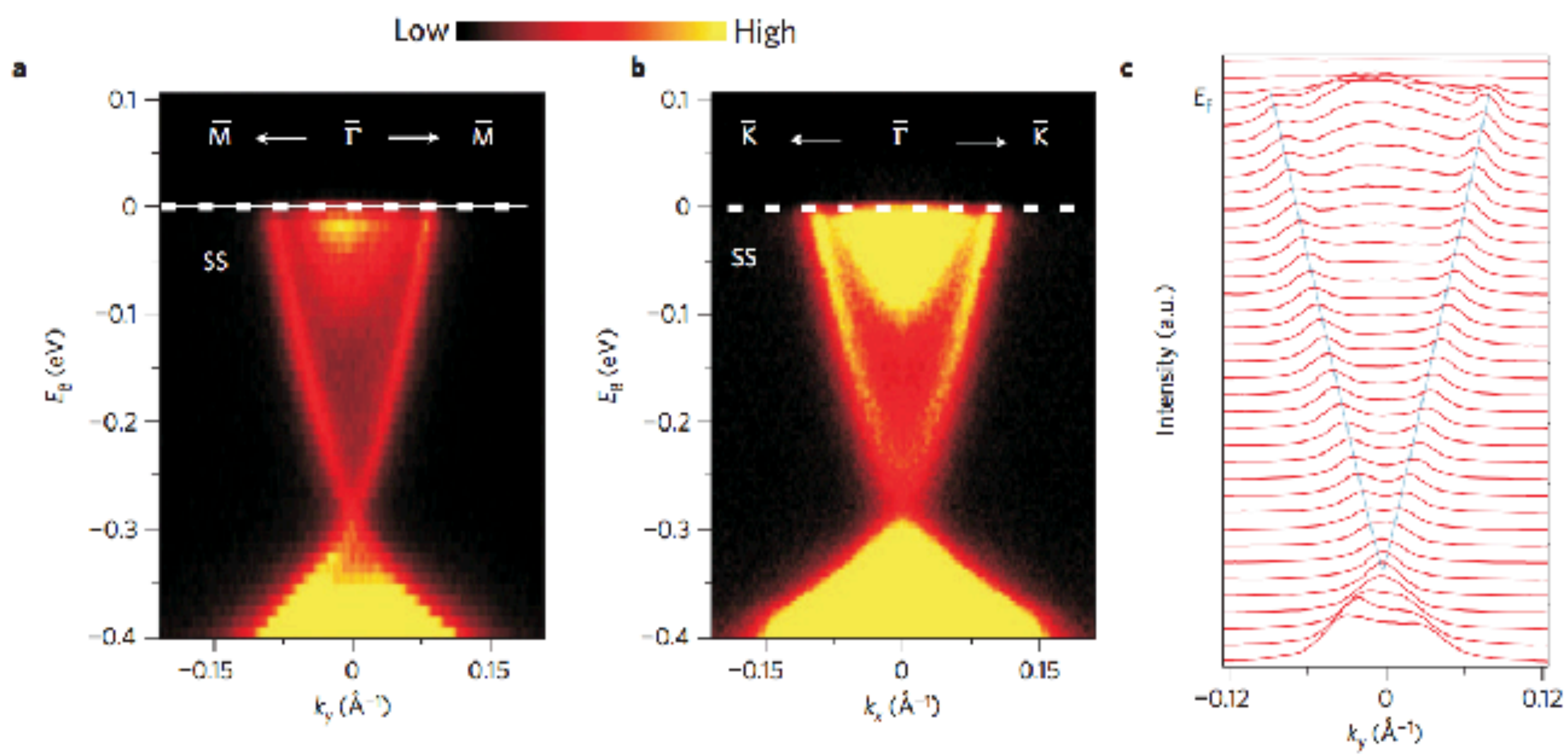
$$E_k = \hbar\omega - E_b - \phi$$

ARPES

LETTERS
PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1274
nature physics

Observation of a large-gap topological-insulator class with a single Dirac cone on the surface

Y. Xia[1,2], D. Qian[1,3], D. Hsieh[1,2], L. Wray[1], A. Pal[1], H. Lin[4], A. Bansil[4], D. Grauer[5], Y. S. Hor[5], R. J. Cava[5] and M. Z. Hasan[1,2,6]★

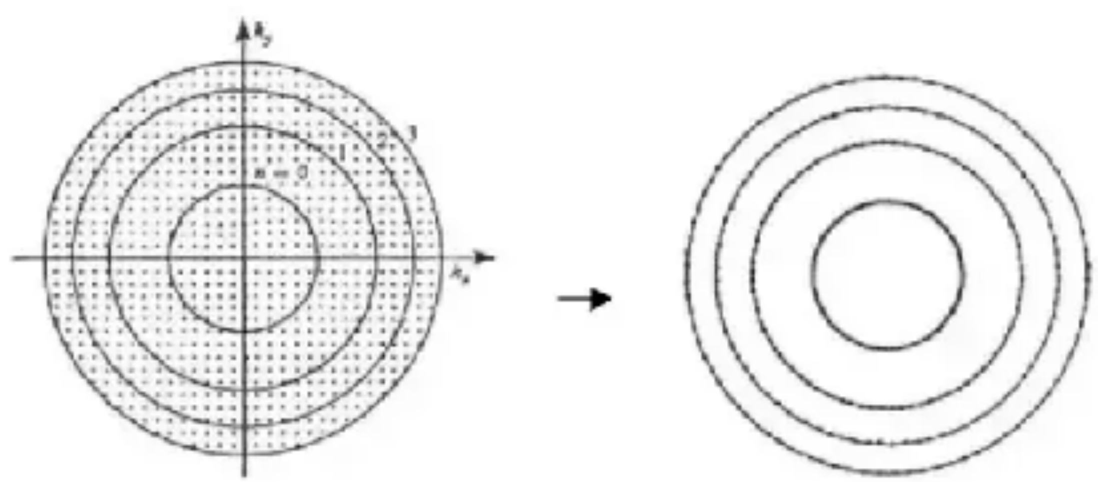


Quantum Oscillations

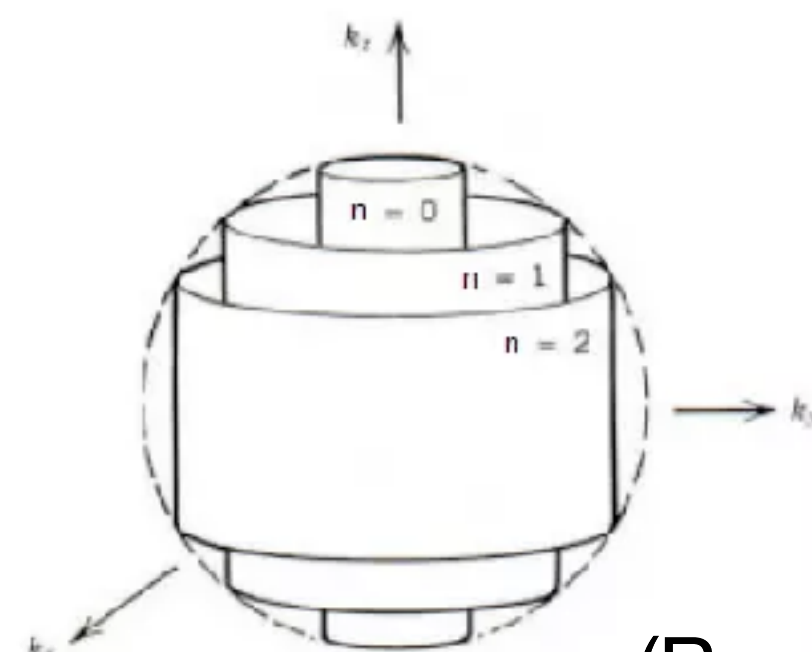
- Oscillation of magnetization (de Haas-van Alphen effect) (Pset 7)
- Oscillation of resistivity (Shubnikov-de Haas effect)
- …

Quantum Oscillations

Landau Levels $$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$



3D: Landau Tubes



Two scenarios:

- Fix particle number
- Fix chemical potential

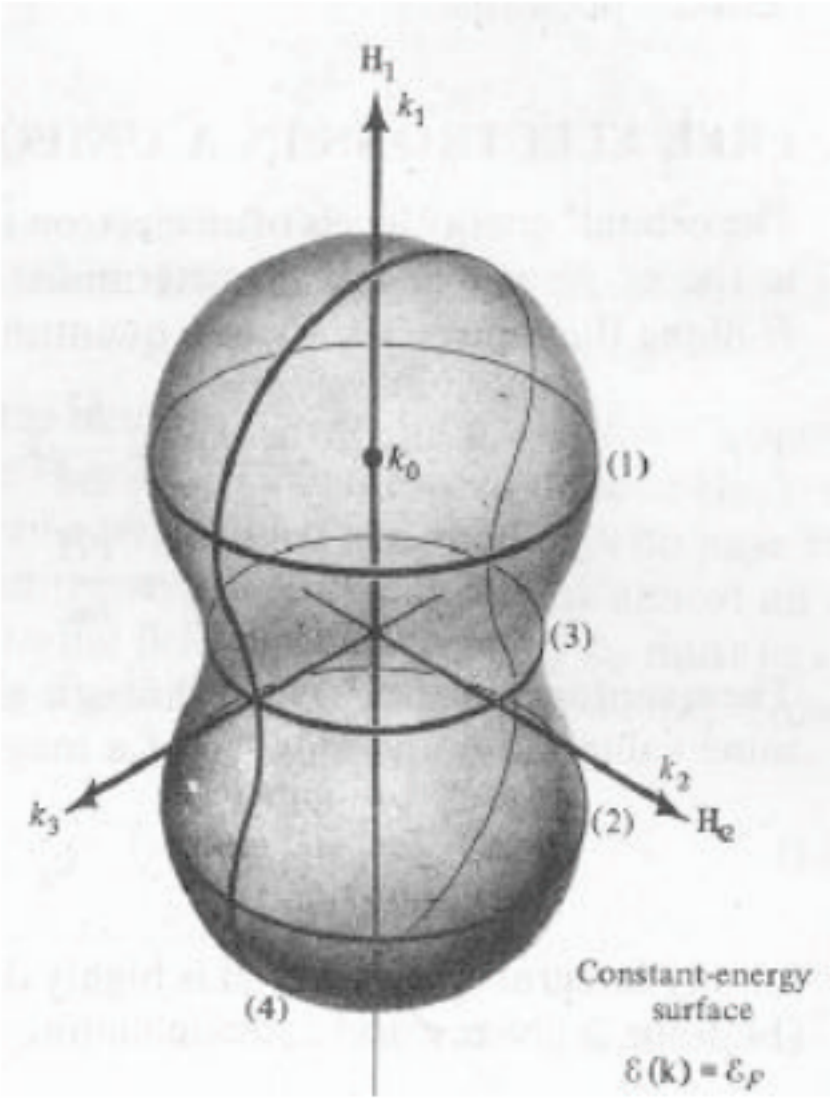
(Rec 5)

Quantum Oscillations

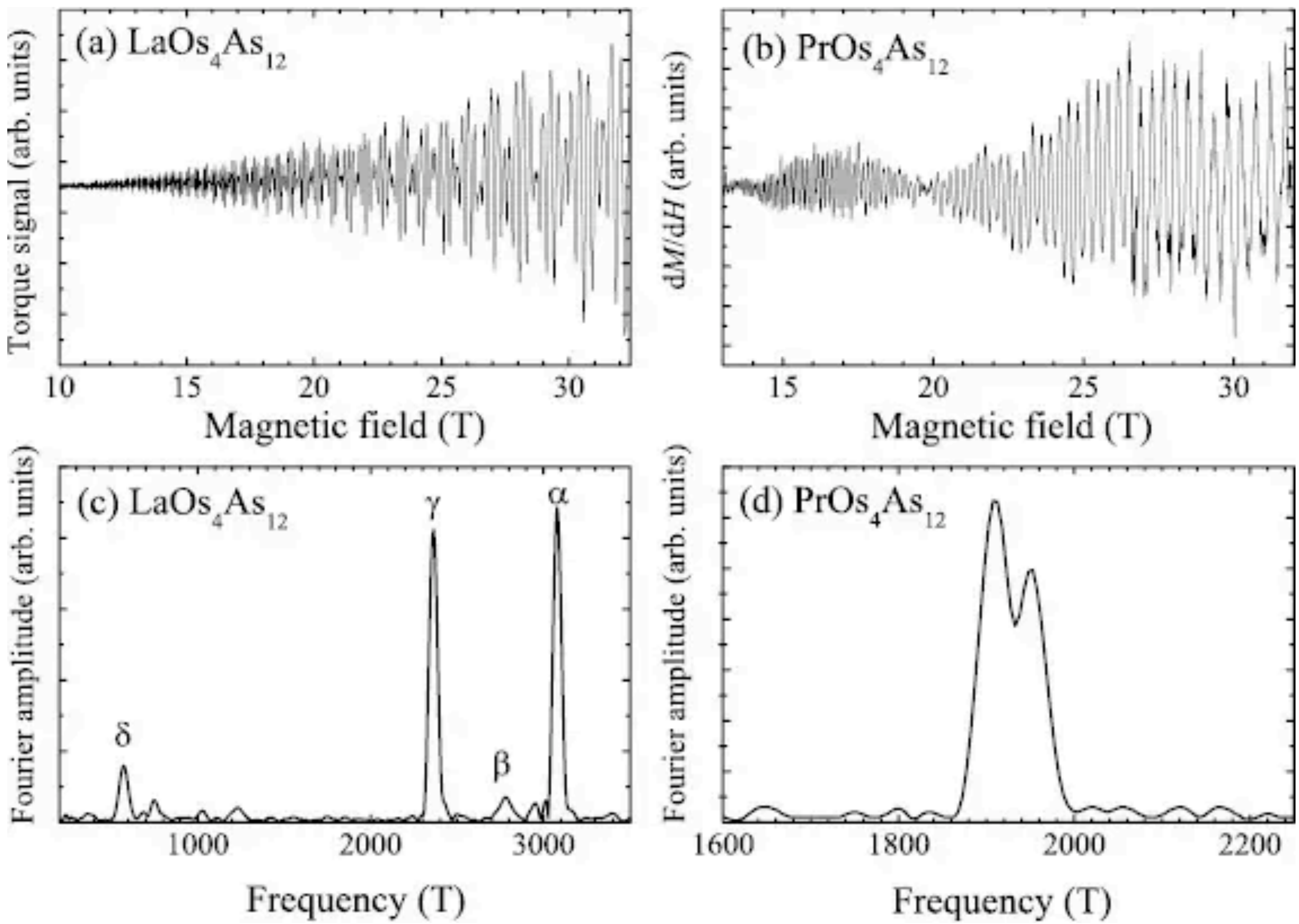
$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi}{A_e}$$

A_e

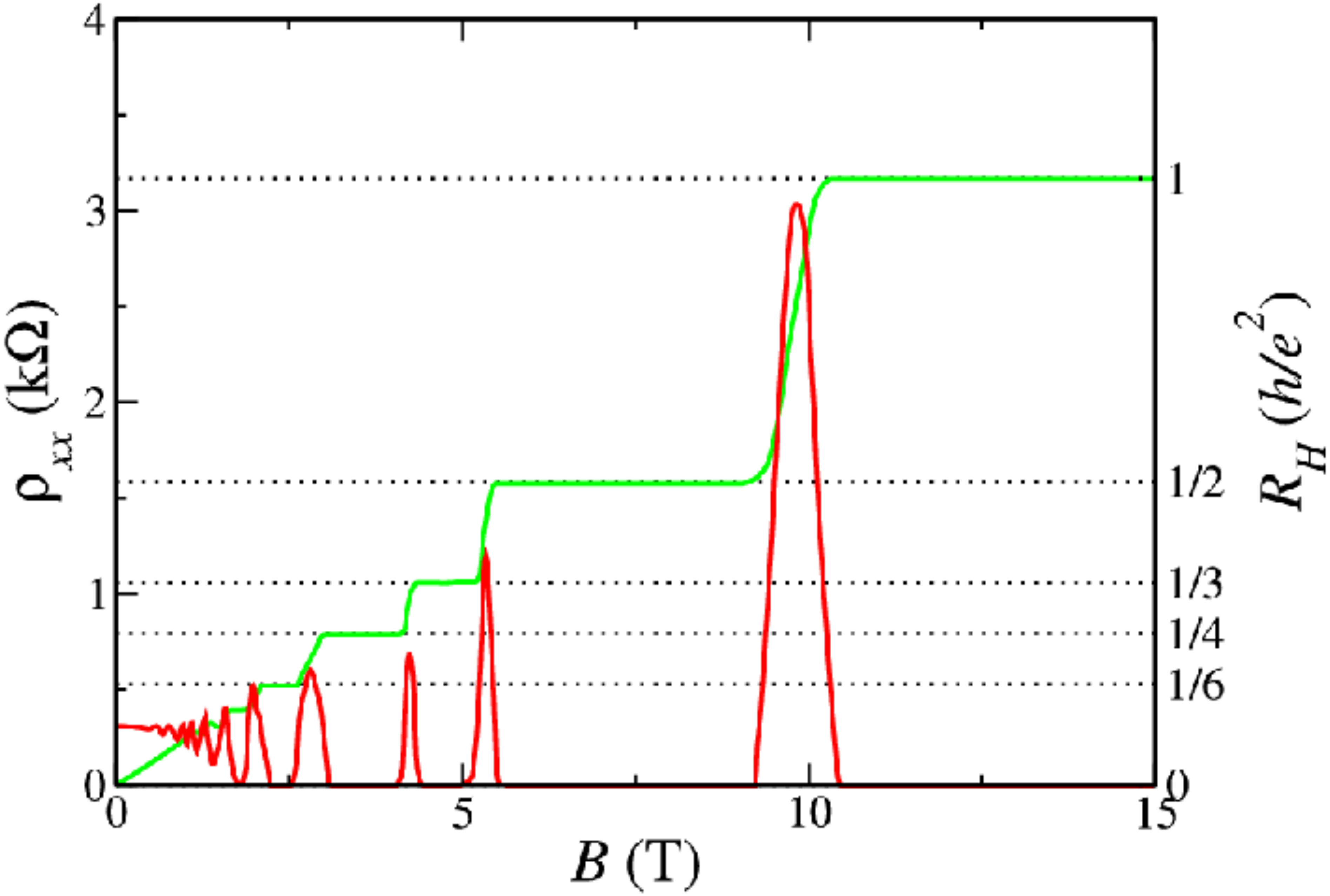
- 2D: Area of the Fermi surface
- 3D: Area of the extremum orbital



Quantum Oscillations

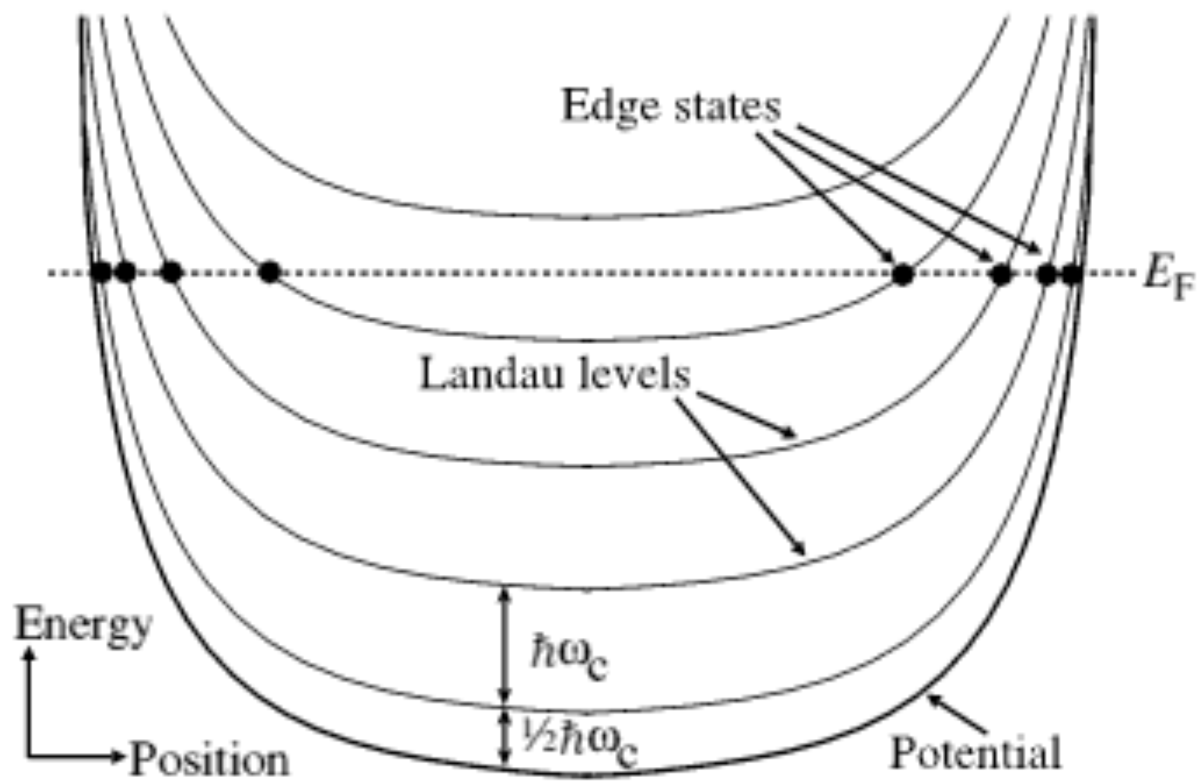


Quantum Hall Effect



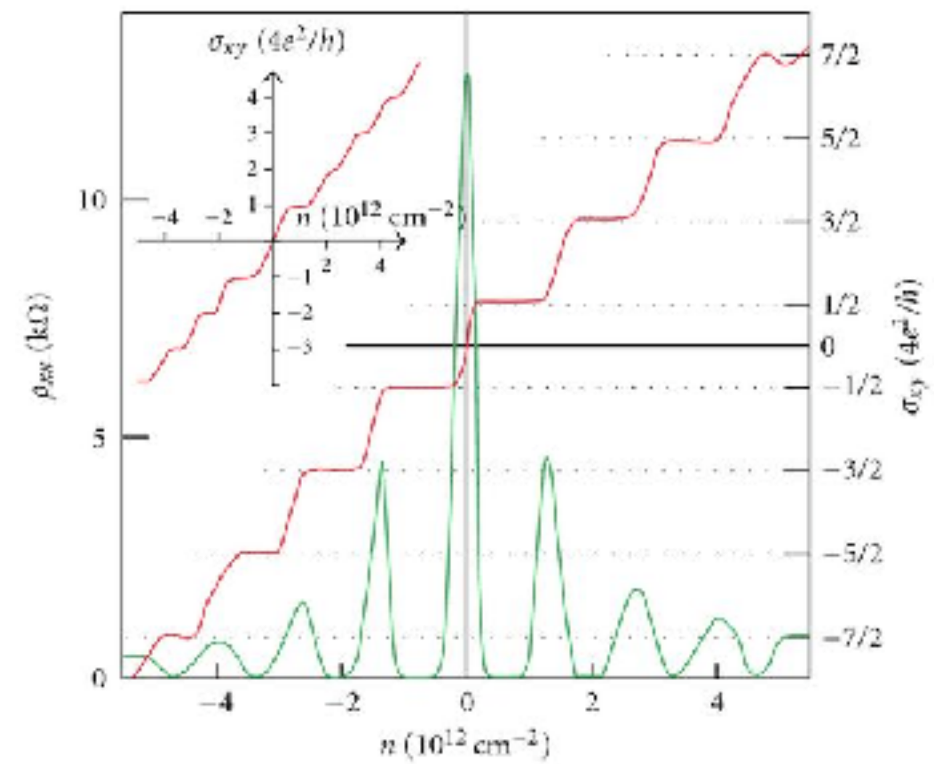
Quantum Hall Effect

- Quantized conductivity comes from edge states
- Impurity is necessary to kill bulk conductance

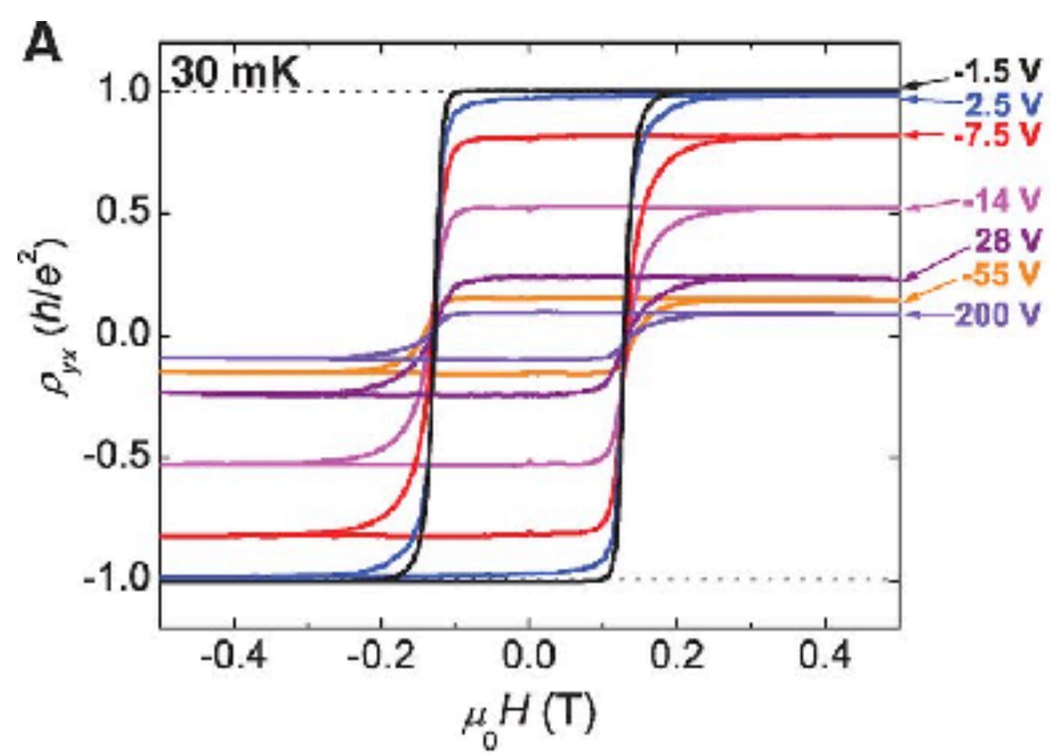


Variance of QHE

- Half-integer quantum Hall effect (graphene)



- Quantum anomalous Hall effect (ferromagnetic topological insulator)



- …

Magnetism

Sources of magnetism:

- Spin magnetic moment: Pauli paramagnetism
- Orbital magnetic moment: Quantum oscillations

Magnetism

Most common magnetic response: $M = \chi H$

- Paramagnetism $\chi > 0$
  - Local moment: Curie paramagnetism $\chi = \frac{C}{T}$
  - Itinerant electron: Pauli paramagnetism
- Diamagnetism $\chi < 0$
  - Landau diamagnetism

Diamagnetism

$$U = -\mathbf{M} \cdot \mathbf{B} = -\chi \mathbf{B} \cdot \mathbf{B}$$

local minimum only when

$$\chi < 0$$

Diamagnetic Levitation
2000 Ig Nobel Prize, Andre Geim

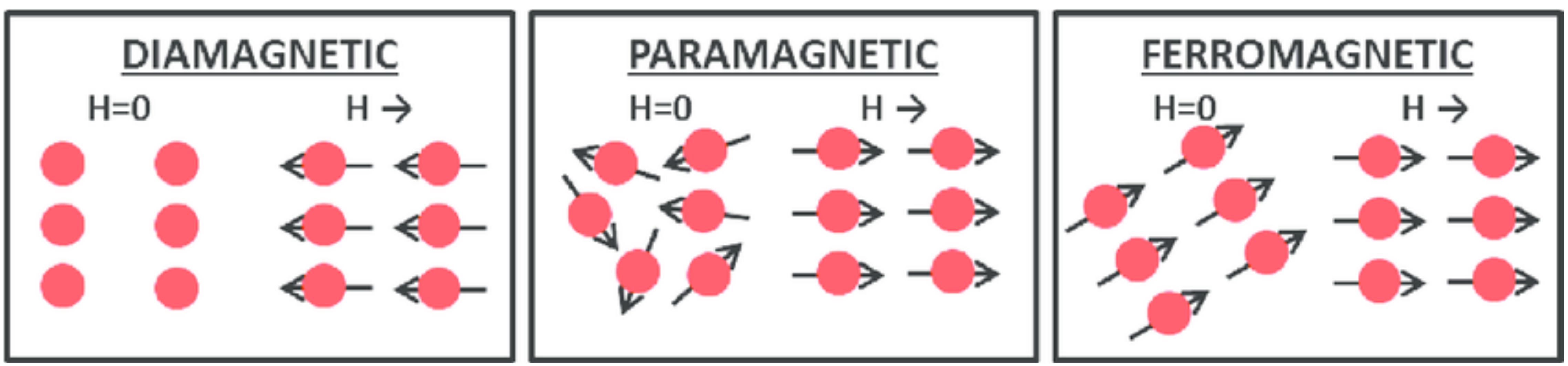
Also 2010 Nobel Prize for graphene

Magnetism

- Paramagnetism
- Diamagnetism
- Ferromagnetism

Ising Model

$$H = J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

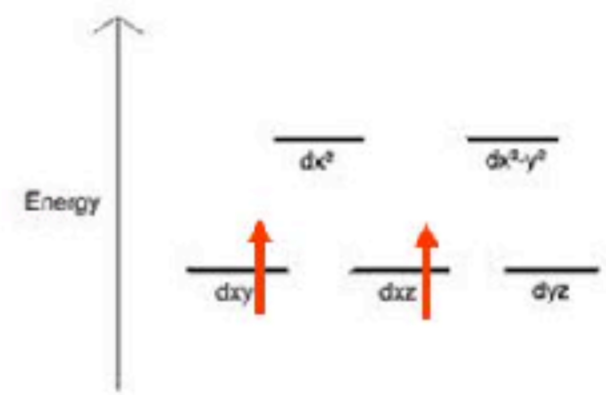


Beyond Band Theory

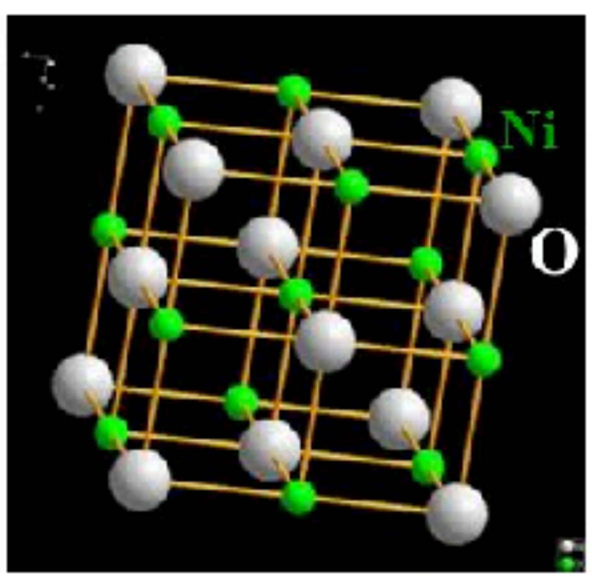
- Mott insulator (el-el interaction)
- Superconductivity (el-ph interaction)
- …

Mott Insulator

TiO- rutile Ti^{2+} $3d^{2}4s^{0}$



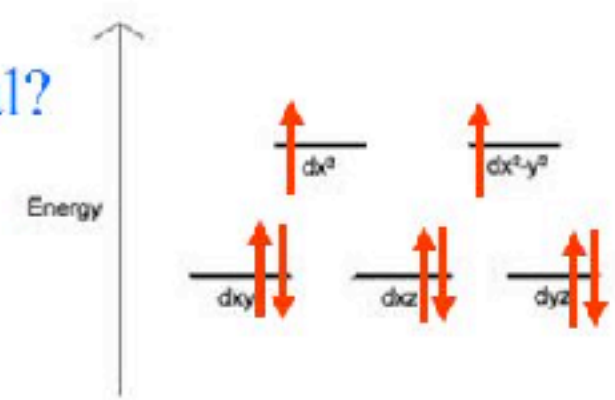
metal



NiO- NaCl structure Ni^{2+} $3d^{8}4s^{0}$

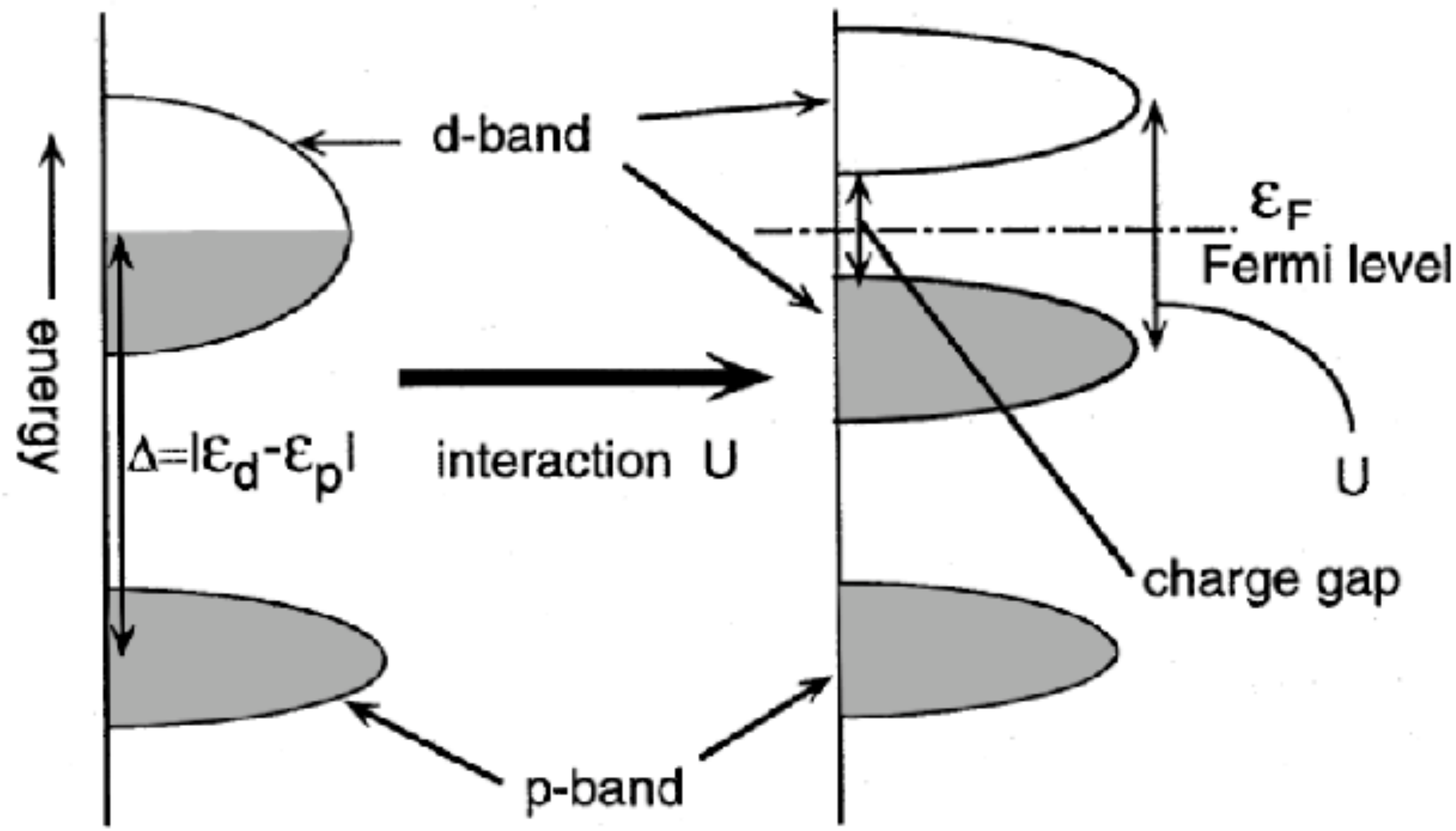
Is insulator!

Why not a metal?



Mott Insulator

Hubbard model $$H = -t \sum_{\langle i,j \rangle, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

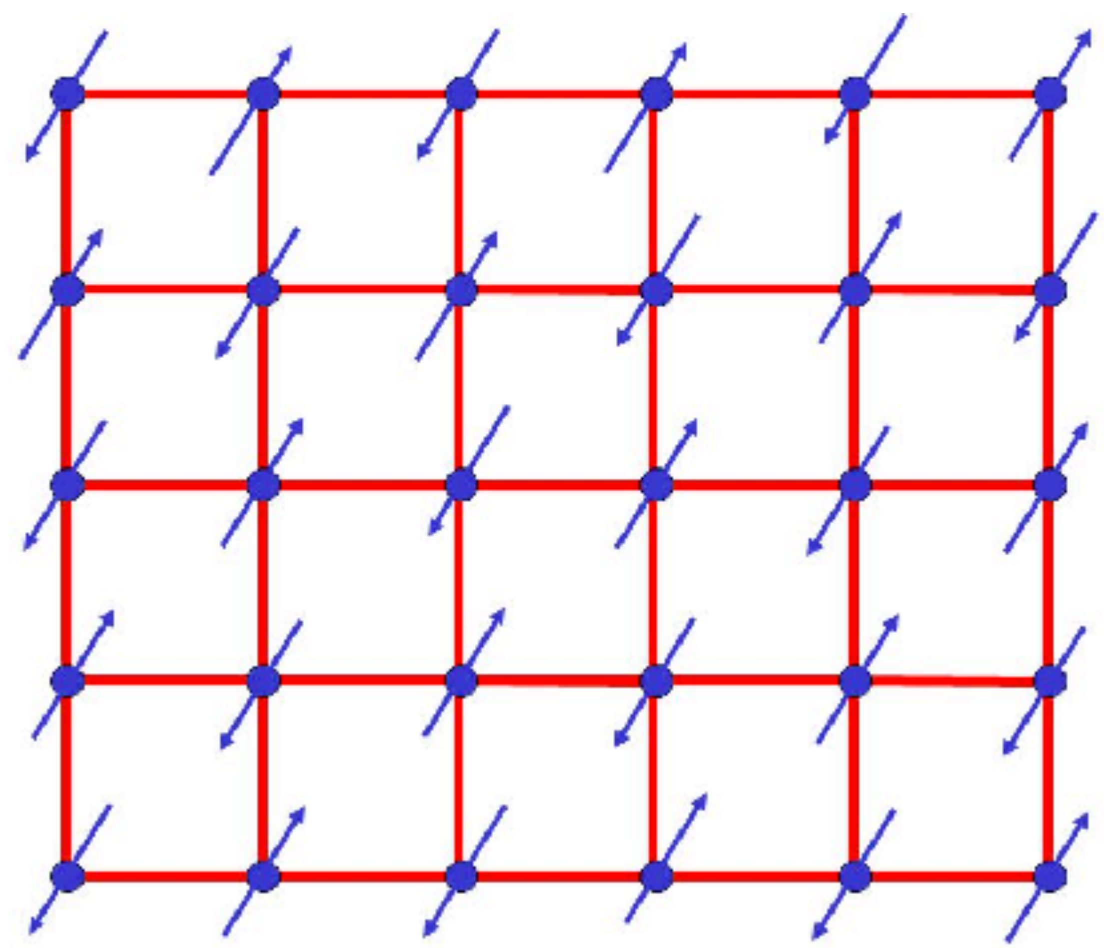


Mott Insulator

$$H = -t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

2nd order perturbation

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j , \quad J \sim \frac{t^2}{U}$$



Neel State

Beyond Band Theory

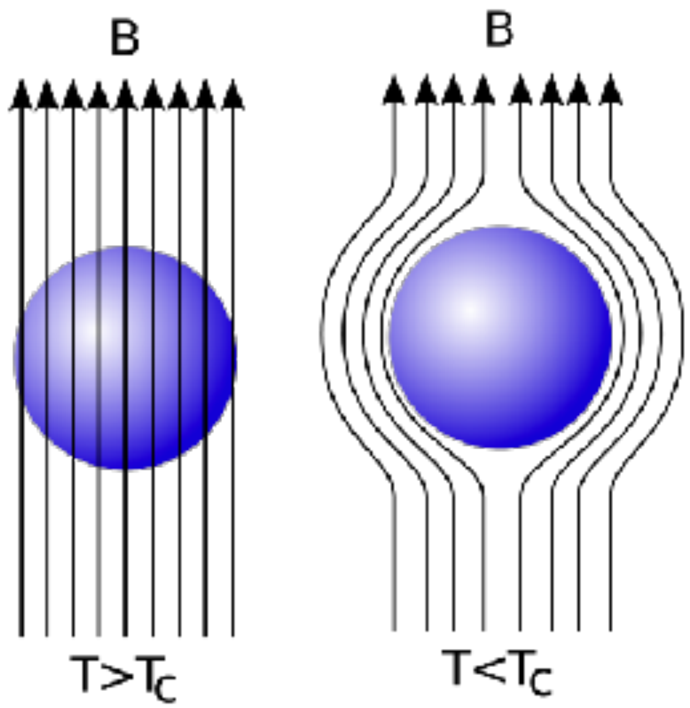
- Mott insulator (el-el interaction)
- Superconductivity (el-ph interaction)
- …

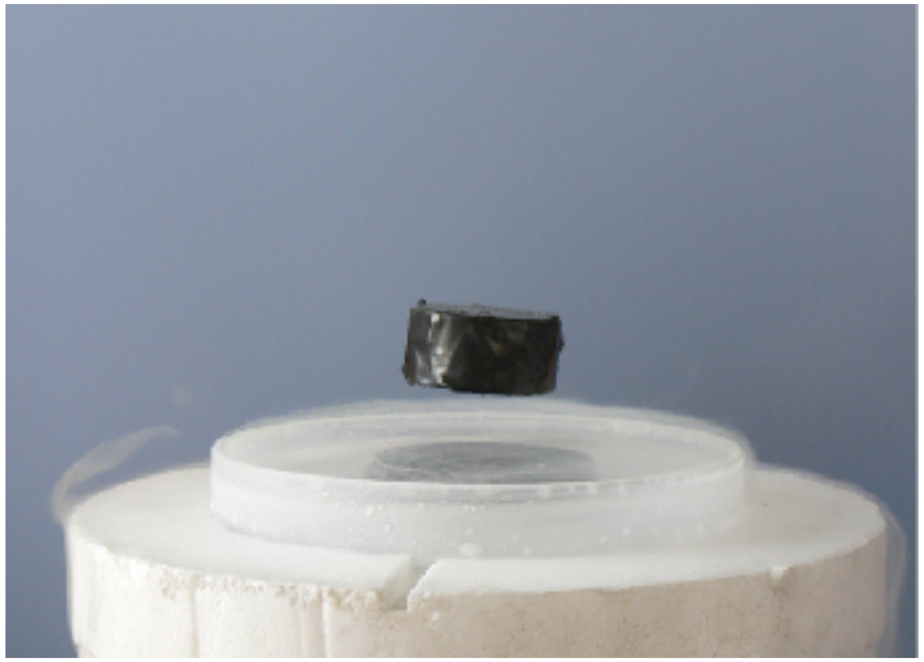
Superconductivity

Defining properties of superconductivity:

- Zero electrical resistivity
- Perfect diamagnetism (Meissner effect)

distinguish superconductors from perfect conductors





Superconductivity

- London equations (1935, Phenomenological, Classical)

$$\frac{\partial \mathbf{J}_s}{\partial t} = \frac{n_s e^2}{m^*}\mathbf{E}$$

$$\nabla \times \mathbf{J}_s = -\frac{n_s e^2}{m^* c}\mathbf{B}$$

- Ginzburg-Landau theory (1950, Phenomenological, Quantum, Rec 8)

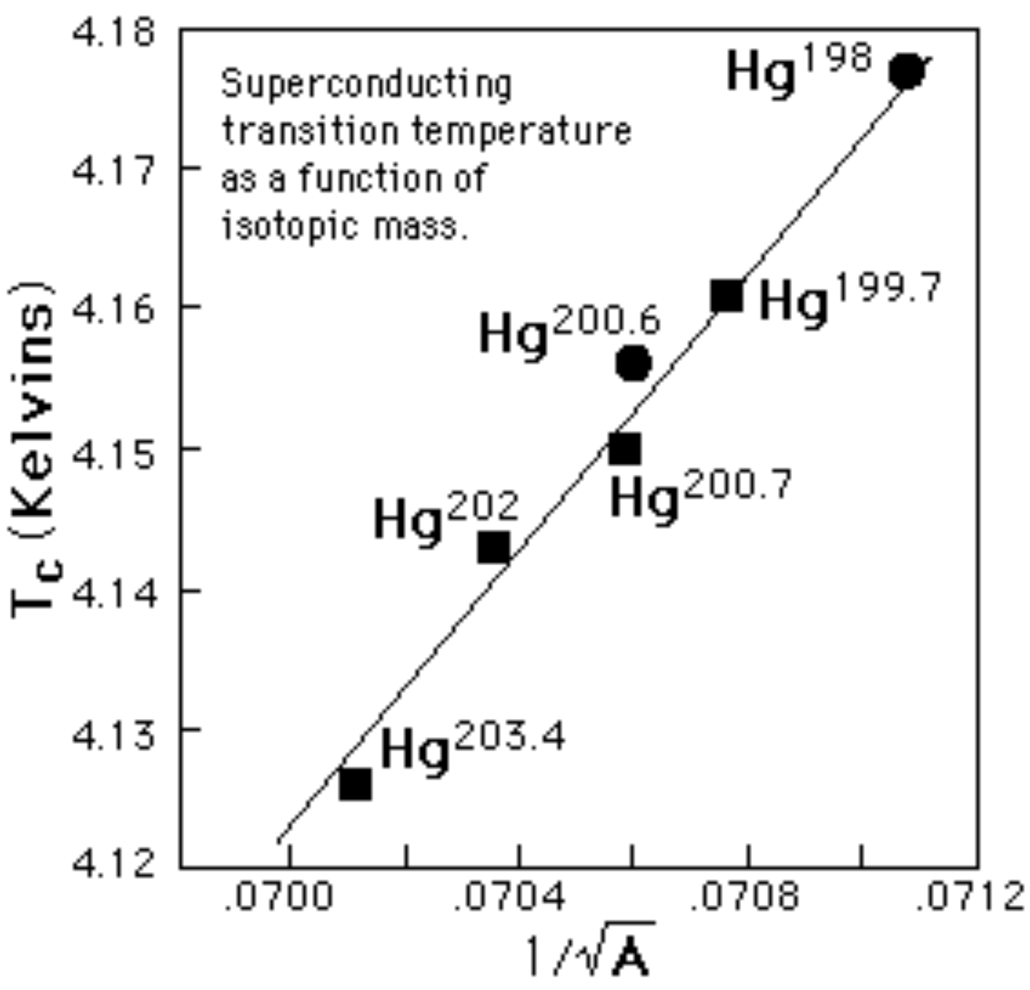
$$f = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\psi\right|^2 + \frac{(\nabla \times \mathbf{A})^2}{8\pi}$$

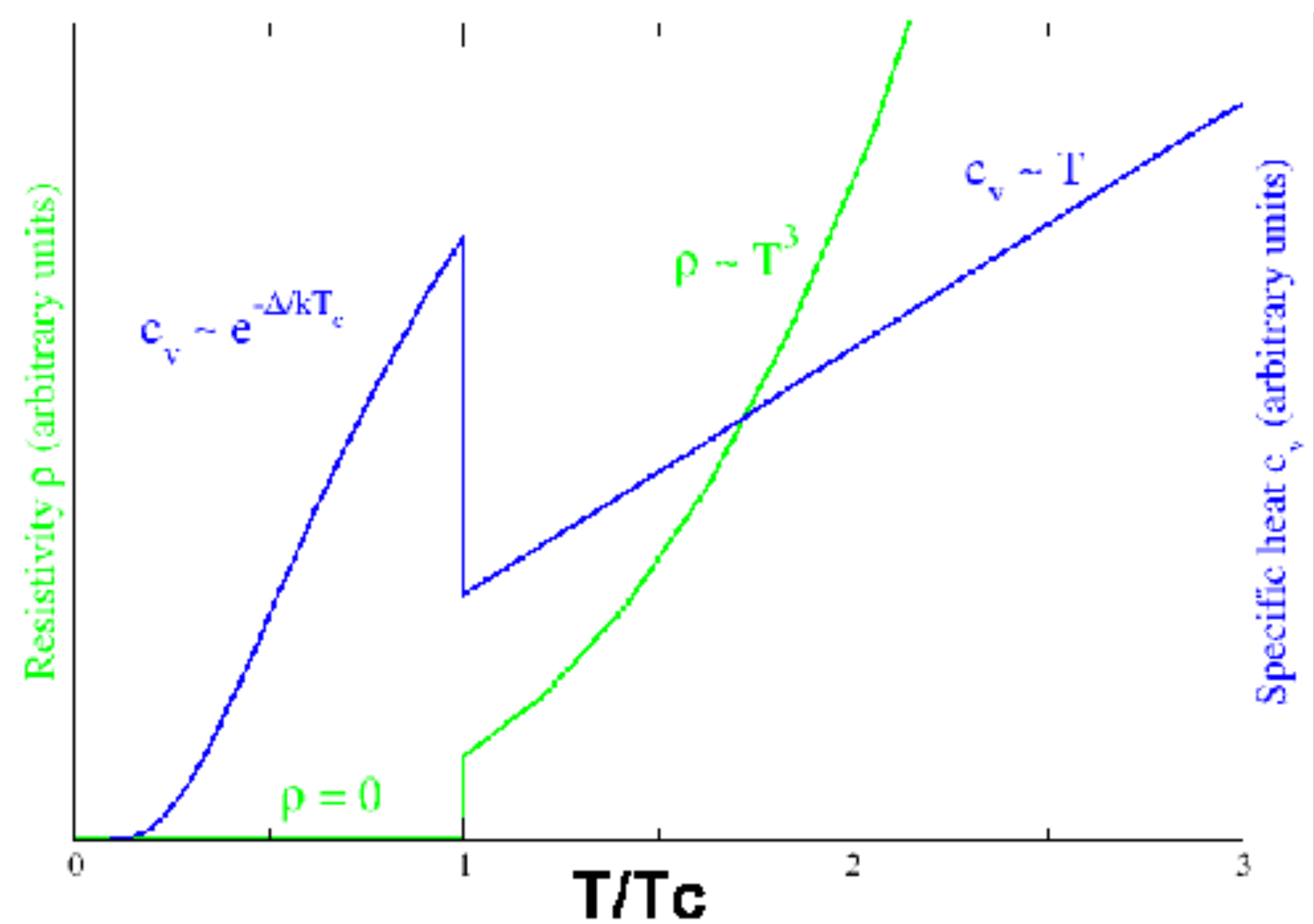
- Cooper pair problem (1956, Pset 8)

- Bardeen-Cooper-Schrieffer theory (1957, Microscopic)

Superconductivity

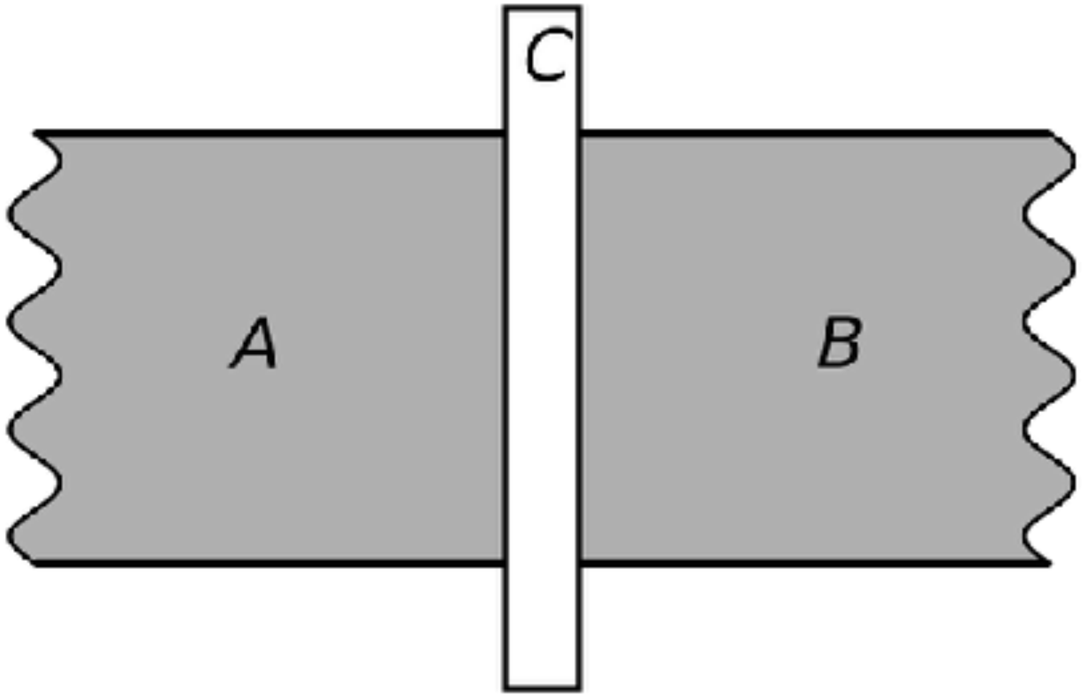
- Isotope effect (1950): related to phonons
- Heat capacity (1956): superconducting state has a gap





Josephson Effect

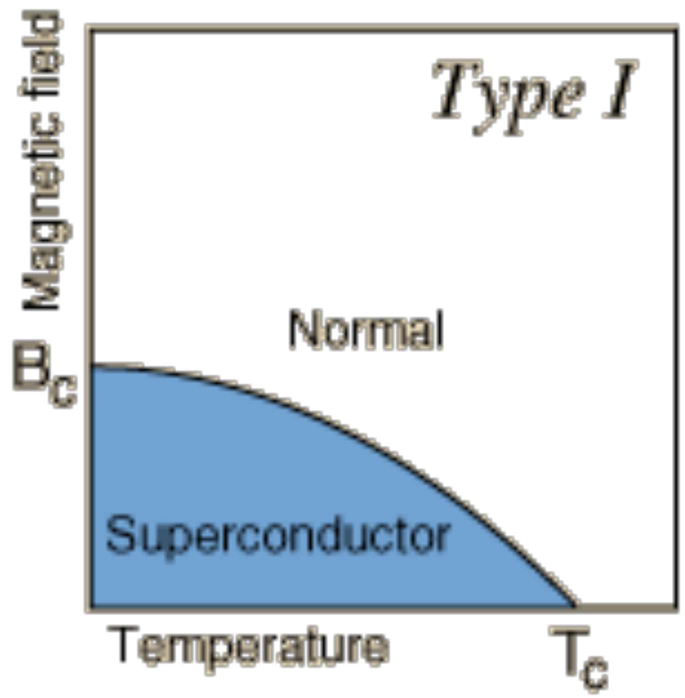
$$U(t) = \frac{\hbar}{2e} \frac{\partial \Delta\phi}{\partial t}$$
$$I(t) = I_c \sin(\Delta\phi(t))$$

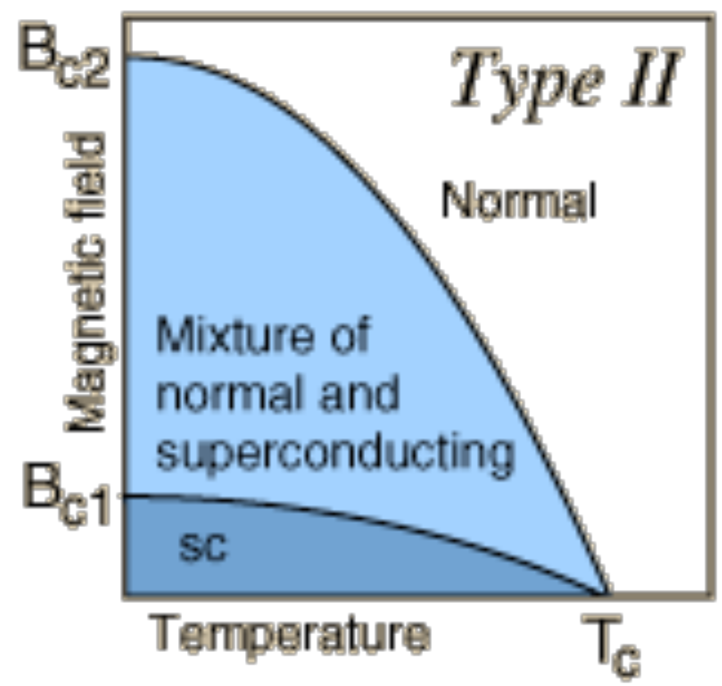


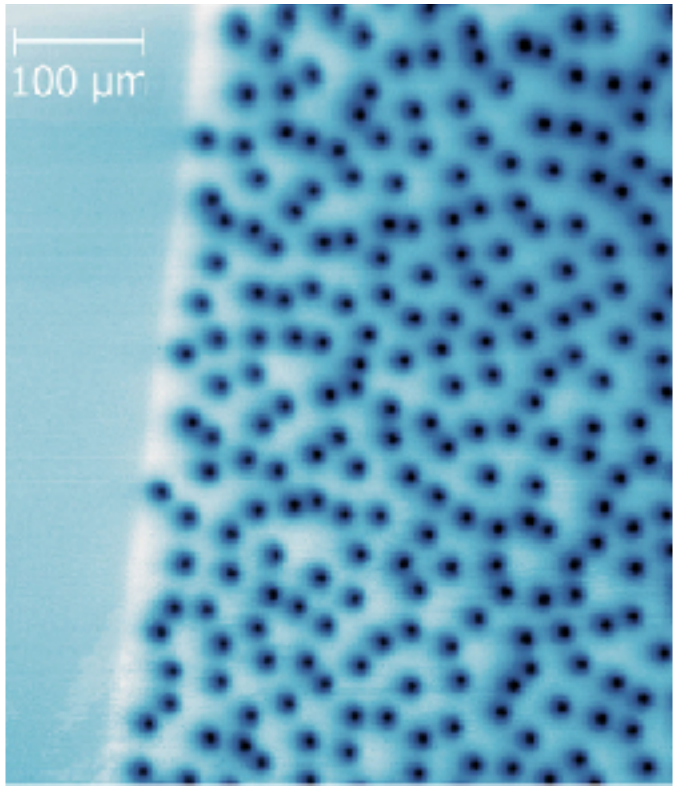
An example of macroscopic quantum phenomenon!

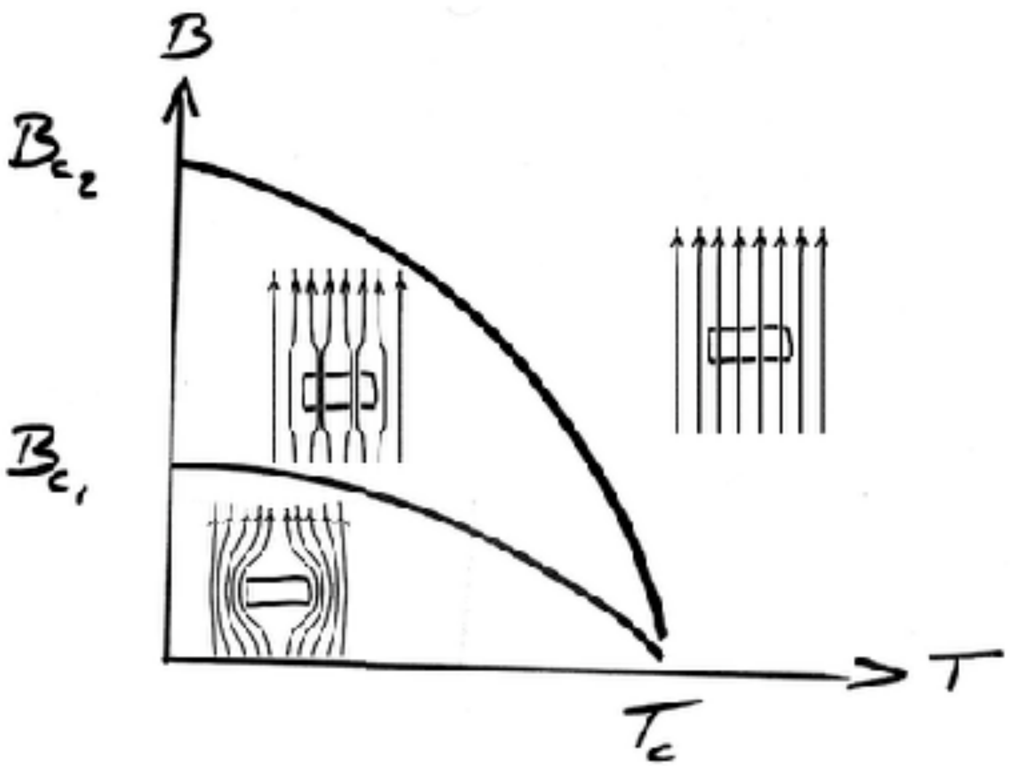
- Bose-Einstein Condensation
- Superfluidity
- …

Superconductivity



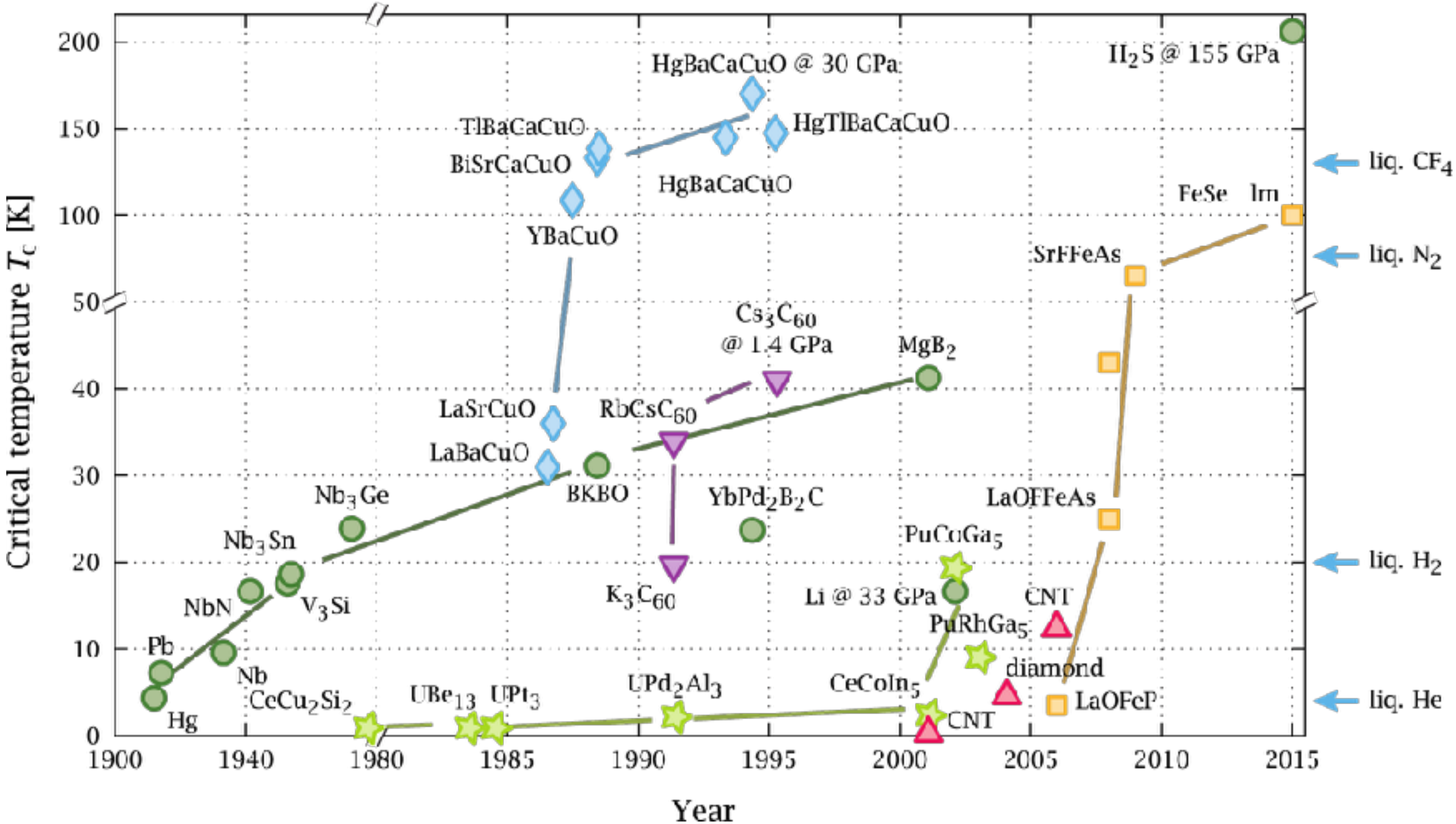






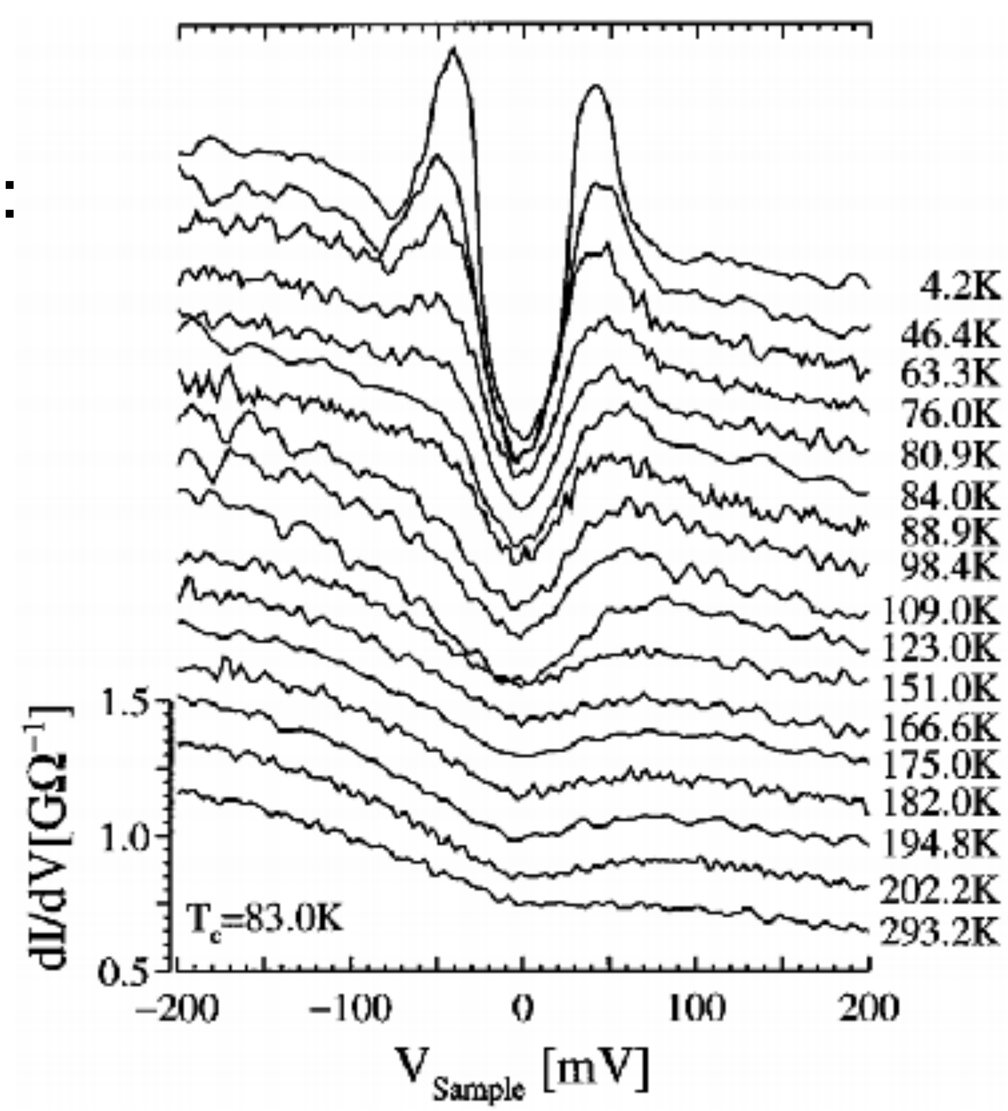
(Rec 8)

High T_c Superconductivity



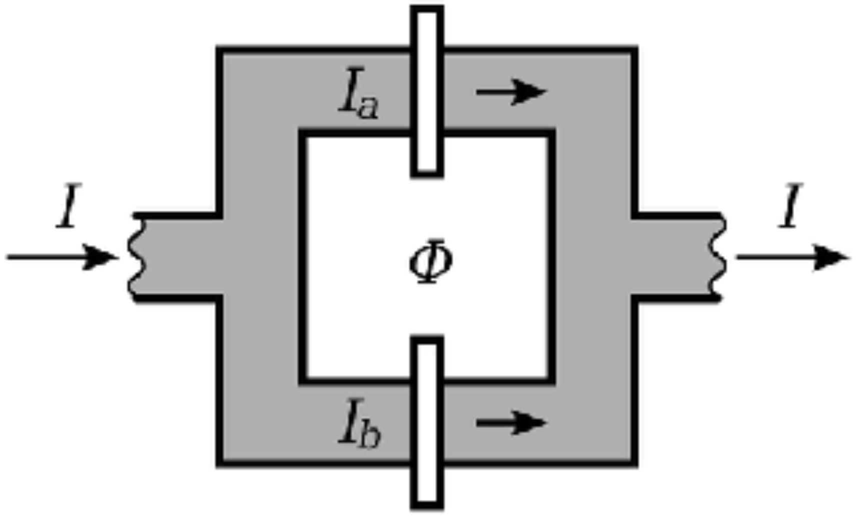
High T_c Superconductivity

- Superconducting phase can be well-described by a 2D, type-II superconductor. Still has Cooper pair.
- But what is the pairing mechanism? Cannot be el-ph interaction as the Debye temperature is so low.
- Even more mysterious pseudogap phase:
    - Gap above critical temperature
    - Linear resistivity in temperature
    - …



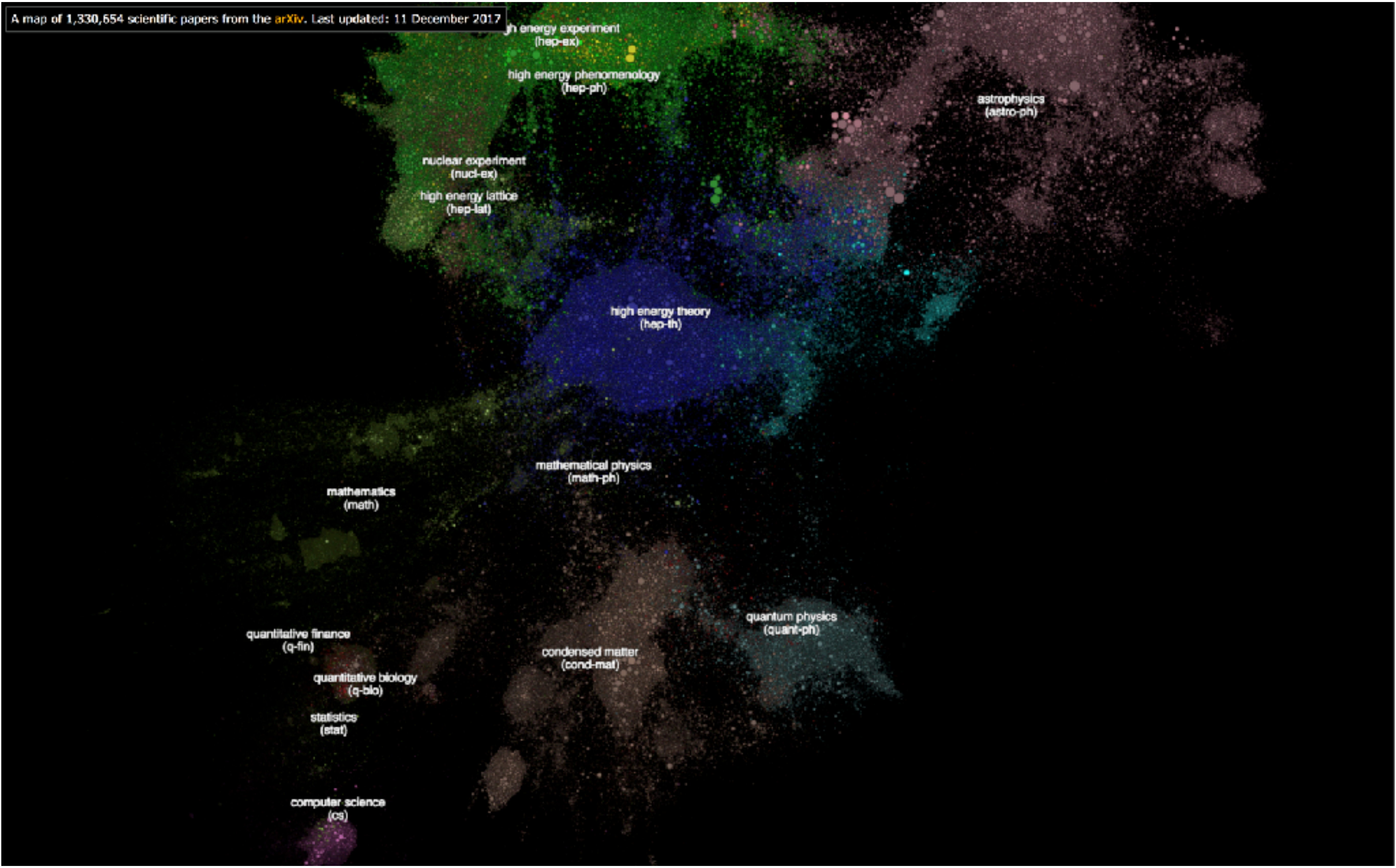
Applications of SC

- Magnetometer (SQUID)
- Superconducting qubits
- Superconducting electromagnets
- …



Beyond Band Theory

- Mott insulator (el-el interaction)
- Superconductivity (el-ph interaction)
- Fractional quantum Hall effect (el-el interaction)
- Kondo effect (el-impurity & el-el interaction)
- …



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More is Different!